Applied Differential Equations 2250
Sample Midterm Exam 1
Tuesday, 9 February 2006
Wednesday, 10 February 2006 arranged

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (Quadrature Equation)
Solve for the general solution $y(x)$ in the equation $y' = xe^{-x} + \csc^2 x + 2 \sec^2 x + \frac{16x^4}{1 + 4x^2}$.
[Integral tables will be supplied for anything other than basic formulae. This sample problem would require no integral table. The exam problem will be shorter.]

Use this page to start your solution. Attach extra pages as needed, then staple.
2. (Separable Equation Test)

The problem

$$y' = x \left( e^{2x} x^{2/3} y e^y + \sin(x) ye^y \right) - x \sin(x) \sin(y) - e^{2x} x^{5/3} \sin(y)$$

may or may not be separable. If it is, then write formulae for $F$, $G$ which decompose the problem as $y' = F(x)G(y)$. Otherwise, explain in detail why it fails to be separable. Do not solve for $y$!

Use this page to start your solution. Attach extra pages as needed, then staple.
3. (Solve a Separable Equation)

Given \( yy' = \frac{6x^3 + 12x}{1 + x} (1 - 4y^2) \),

(a) Find all equilibrium solutions,

(b) Find the non-equilibrium solution in implicit form.

Do not solve for \( y \) explicitly.

Use this page to start your solution. Attach extra pages as needed, then staple.
4. (Linear Equations)
   (a) Solve $2v'(t) = -64 + \frac{4}{t+1}v(t), \ v(0) = 3$. Show all integrating factor steps.
   (b) Using the answer $v(t)$ from (a), solve $y'(t) = v(t), \ y(0) = 2$. Show all quadrature steps.
5. (Stability)
   (a) Draw a phase line diagram for the differential equation

   \[ \frac{dx}{dt} = (2 - 5x)^3(1 - 2x)(1 - 4x^2). \]

   Expected in the diagram are equilibrium points and signs of \( x' \) (or flow direction markers < and >).

   (b) Draw a phase diagram using the phase line diagram of (a). Add these labels as appropriate: funnel, spout, node, stable, unstable. Show at least 10 threaded curves. A direction field is not required.