1.3 #33 If $c \neq 0$ verify that the function $y(x) = \frac{x}{cx-1}$ satisfies the differential equation $x^2 y' + y^2 = 0$ if $x \neq \frac{1}{c}$. Sketch a variety of such solution curves for different values of $c$.

$$y'(x) = \frac{(cx-1)-x\cdot c}{(cx-1)^2}$$

take derivative of $y$ and plug into the diff eq

$$\frac{x^2(cx-1)-x^2c}{(cx-1)^2} + \left(\frac{x}{cx-1}\right)^2 = 0$$

$$\frac{c x^2 - x^2 - x'c}{(cx-1)^2} = -\frac{x^2}{(cx-1)^2}$$

$$-x^2 = -x^2$$

both sides of equation are equal

Determine in terms of $a$ and $b$ how many different solutions the initial value problem $x^2 y' + y^2 = 0$, $y(a) = b$ has.

$$x^2 \frac{dy}{dx} = -y^2$$

$$\frac{dy}{dx} = -\frac{y^2}{x^2}$$

solve for $y$ by using separation of variables

$$\int \frac{dy}{y^2} = \int \frac{dx}{x^2}$$
\[-\frac{1}{y} = \frac{1}{x} + c\]

\[y = -\frac{1}{\frac{1}{x} + c}\]

\[b = -\frac{1}{\frac{1}{a} + c}\]

If \(a = 0\) then \(b\) must equal \(0\) (infinite number of solutions)

If \(a = 0\) and \(b \neq 0\) then there's no solution

Otherwise there is a unique solution

Answer: check with the book