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1.3 #33 If $c \neq 0$ verify that the function $y(x) = \frac{x}{cx-1}$ satisfies the differential equation $x^2y' + y^2 = 0$ if $x \neq \frac{1}{c}$. Sketch a variety of such solution curves for different values of c.

$$y'(x) = \frac{(cx-1) - xc}{(cx-1)^2}$$

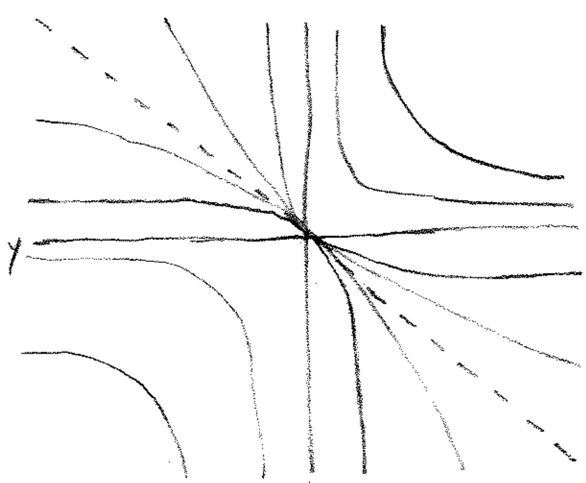
take derivative of y and plug into the dif eq

$$\frac{x^2(cx-1) - x^3c}{(cx-1)^2} + \left(\frac{x}{cx-1}\right)^2 = 0$$

$$\frac{cx^3 - x^2 - x^3c}{(cx-1)^2} = -\frac{x^2}{(cx-1)^2}$$

$$-x^2 = -x^2$$

both sides of equation are equal



Determine in terms of a and b how many different solutions the initial value problem $x^2y' + y^2 = 0$, $y(a) = b$ has.

$$x^2 \frac{dy}{dx} = -y^2$$

$$\frac{dy}{dx} = \frac{-y^2}{x^2}$$

solve for y by using separation of variables

$$\int \frac{dy}{y^2} = \int -\frac{dx}{x^2}$$

$$-\frac{1}{y} = \frac{1}{x} + C$$

$$y = -\frac{1}{\frac{1}{x} + C}$$

$$b = -\frac{1}{\frac{1}{a} + C}$$

if $a=0$ then b must equal 0 (infinite number of solutions)

if $a=0$ and $b \neq 0$ then there's no solution

otherwise there is a unique solution

answer check with the book