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Problem #:10. Section: 4.3
Class: MATH2280-002
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Use Runge-Kutta method with step sizes h=0.1 and h=0.05 to approximate to five decimal places the values x(1) and y(1) of differential equations: x'=x+2y, y'=x+e^(-t); x(0)=0, y(0)=0. Compare the approximations with the actual values of exact solution (Note: Book has an error in their exact solution so I used DSolve to find correct solution)

Setting initial conditions with h=0.1.
Defining components of vector function and vector function itself.

```
ti = 0; X = {0, 0};
h = 0.1;
f1[t_, {x_, y_}] := x + 2 y;
f2[t_, {x_, y_}] := x + e^-t;
F[t_, x_] := (f1[t, x]
              f2[t, x]);
```

Defining Runge-Kutta algorithm from Eq. 11 & 12

```
rk4[n_] := While[ti < n,
      K1 = Flatten[F[ti, X], 1];
      K2 = Flatten[F[ti + 1/2 h, X + 1/2 h K1], 1];
      K3 = Flatten[F[ti + 1/2 h, X + 1/2 h K2], 1];
      K4 = Flatten[F[ti + h, X + h K3], 1];
      X1 = X + h/6 (K1 + 2 K2 + 2 K3 + K4);
      X = X1;
      tf = ti + h;
      ti = tf;]
rk4[1];
X
 ={1.31497663979,1.02536729236}
```

Approximation with h=0.1

Reinitializing variables and setting h=0.05

Reusing Runge-Kutta algorithm.

```
ti=0; X={0,0};
h=0.05;
rk4[1]
X
 ={1.31500633617,1.02538258148}
```

Approximation with h=0.05

Due to error in the book, finding exact solution with DSolve (books solution is:
x(t)=1/9(2e^(2t)-2e^(-t)+6te^(-t))
correct solution is:
x(t)=1/9(2e^(2t)-2e^(-t)-6te^(-t)),
y(t) is correct in the book.)

```
FullSimplify[DSolve[{x'[t] == x[t] + 2 y[t], y'[t] == x[t] + e^-t, x[0] == 0, y[0] == 0},
  {x[t], y[t]}, t]]
```

$$\left\{\left\{x[t] \to \frac{2}{9} e^{-t}\left(-1+e^{3t}-3t\right),\ y[t] \to \frac{1}{9} e^{-t}\left(-1+e^{3t}+6t\right)\right\}\right\}$$

```
t = 1;
```

$$N\left[\left\{\frac{2}{9} e^{-t}\left(-1+e^{3t}-3t\right),\ \frac{1}{9} e^{-t}\left(-1+e^{3t}+6t\right)\right\}\right]$$

Exact values.

```
{1.31500851872,1.02538370053}
```

Answer Check: B.O.B

$b3 = \frac{2\,a2 + b2}{\sqrt{3}};$

$\text{Solve}[-2\,a3 - \sqrt{3}\,b2 - b3 == 0,\ b2]$

$\{\{b2 \to -\frac{\sqrt{3}\,a2 + 3\,a3}{2\sqrt{3}}\}\}$

$b2 = \text{Simplify}[-\frac{\sqrt{3}\,a2 + 3\,a3}{2\sqrt{3}}]$

b2 in terms of a2 and a3.

$\frac{1}{2}\,(-a2 - \sqrt{3}\,a3)$

$b3 = \text{Simplify}[\frac{2\,a2 + b2}{\sqrt{3}}]$

b3 in terms of a2 and a3.

$\frac{1}{2}\,(\sqrt{3}\,a2 - a3)$

$\text{Simplify}[-2\,b2 - c2 + \sqrt{3}\,c3]$

0

$\text{Simplify}[-2\,b3 - \sqrt{3}\,c2 - c3]$

0

Local answer check. Should comeback with 0, and it does.

Sub known values to find a1, a2 and a3

x1[0]==100

a1+a2==100

x2[0]==0

$a1 + \frac{1}{2}\,(-a2 - \sqrt{3}\,a3) == 0$

x3[0]==0

$a1 + \frac{1}{2}\,(-a2 + \sqrt{3}\,a3) == 0$

$\text{TableForm}[\text{RowReduce}[\begin{pmatrix} 1 & 1 & 0 & 100 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{pmatrix}]]$

1	0	0	$\frac{100}{3}$
0	1	0	$\frac{200}{3}$
0	0	1	0

$a1 = \frac{100}{3};\ a2 = \frac{200}{3};\ a3 = 0;$

Values of a1, a2 and a3

FullSimplify[x1[t]]

$\frac{100}{3}\left(1 + 2\,e^{-3t/20}\,\text{Cos}[\frac{\sqrt{3}\,t}{20}]\right)$

Solution to the problem:
x1[t]

FullSimplify[x2[t]]

$\frac{100}{3}\left(1 + e^{-3t/20}\left(-\text{Cos}[\frac{\sqrt{3}\,t}{20}] + \sqrt{3}\,\text{Sin}[\frac{\sqrt{3}\,t}{20}]\right)\right)$

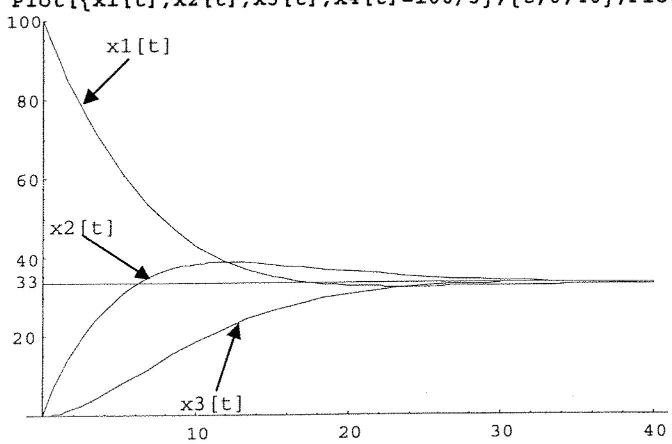
x2[t]

FullSimplify[x3[t]]

$\frac{100}{3}\left(1 - e^{-3t/20}\left(\text{Cos}[\frac{\sqrt{3}\,t}{20}] + \sqrt{3}\,\text{Sin}[\frac{\sqrt{3}\,t}{20}]\right)\right)$

x3[t] all ok

Plot[{x1[t],x2[t],x3[t],x4[t]=100/3},{t,0,40},PlotRange→{0,100}]



Plot of the amount of salt in each tank for t=0 to t=40. Graph shows that as t approaches infinity amount of salt in each tank approaches 33.(3) lb.

Answer Check :

Simplify[10x1'[t]==-x1[t]+x3[t]]

True

Simplify[10x2'[t]==x1[t]-x2[t]]

True

Simplify[10x3'[t]==x2[t]-x3[t]]

True

Answer check by substituting solutions into original problem. Should comeback with True, and it does.