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Problem #:34. Section: 4.2
Class: MATH2280-002
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Three 100-gal brine tanks are connected as indicated in Fig. 4.1.13 of Section 4.1. Assume that the fist tank initially contains 100 lb of salt, whereas the other two are filled with fresh water. Find the amounts of salt in each of the three tanks at time t.

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Clear[a1, a2, a3, b1, b2, b3, c1, c2, c3, t, x1, x2, x3]
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10 x1'[t] == -x1[t] + x3[t];
10 x2'[t] == x1[t] - x2[t];
10 x3'[t] == x2[t] - x3[t];
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Equations from Problem 26 pg. 251

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(10 D + 1) x1[t]                 + (-1) x3[t]       == 0;
(-1) x1[t]      + (10 D + 1) x2[t]                  == 0;
                 (-1) x2[t]      + (10 D + 1) x3[t] == 0;
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Rearranged and grouped

$$\text{Det}\left[\begin{pmatrix} 10r+1 & 0 & -1 \\ -1 & 10r+1 & 0 \\ 0 & -1 & 10r+1 \end{pmatrix}\right]$$

$$30r + 300r^2 + 1000r^3$$

Deriving Char. Eq.

$$\text{Solve}[30r + 300r^2 + 1000r^3 == 0, r]$$

$$\left\{\{r \to 0\}, \left\{r \to \frac{1}{20}\left(-3 - i\sqrt{3}\right)\right\}, \left\{r \to \frac{1}{20}\left(-3 + i\sqrt{3}\right)\right\}\right\}$$

Roots of Char. Eq.

$$\text{x1[t\_]} := a1 + e^{-\frac{3t}{20}}\left(a2\,\text{Cos}\left[\frac{\sqrt{3}}{20}t\right] + a3\,\text{Sin}\left[\frac{\sqrt{3}}{20}t\right]\right);$$

$$\text{x2[t\_]} := b1 + e^{-\frac{3t}{20}}\left(b2\,\text{Cos}\left[\frac{\sqrt{3}}{20}t\right] + b3\,\text{Sin}\left[\frac{\sqrt{3}}{20}t\right]\right);$$

$$\text{x3[t\_]} := c1 + e^{-\frac{3t}{20}}\left(c2\,\text{Cos}\left[\frac{\sqrt{3}}{20}t\right] + c3\,\text{Sin}\left[\frac{\sqrt{3}}{20}t\right]\right);$$

General solution with some of the coefficients being not linearly independent.

$$\text{Collect}\left[\text{Simplify}[10\,x1'[t] + x1[t] - x3[t]], \left\{\text{Cos}\left[\frac{\sqrt{3}\,t}{20}\right], \text{Sin}\left[\frac{\sqrt{3}\,t}{20}\right]\right\}\right] == 0$$

$$a1 - c1 + \frac{1}{2}\left(-a2 + \sqrt{3}\,a3 - 2\,c2\right)e^{-3t/20}\,\text{Cos}\left[\frac{\sqrt{3}\,t}{20}\right] + \frac{1}{2}\left(-\sqrt{3}\,a2 - a3 - 2\,c3\right)e^{-3t/20}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{20}\right] == 0$$

Substitute general solution into original equations to determine dependences between coefficients.

$$\text{Collect}\left[\text{Simplify}[10\,x2'[t] - x1[t] + x2[t]], \left\{\text{Cos}\left[\frac{\sqrt{3}\,t}{20}\right], \text{Sin}\left[\frac{\sqrt{3}\,t}{20}\right]\right\}\right] == 0$$

$$-a1 + b1 + \frac{1}{2}\left(-2\,a2 - b2 + \sqrt{3}\,b3\right)e^{-3t/20}\,\text{Cos}\left[\frac{\sqrt{3}\,t}{20}\right] + \frac{1}{2}\left(-2\,a3 - \sqrt{3}\,b2 - b3\right)e^{-3t/20}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{20}\right] == 0$$

$$\text{Collect}\left[\text{Simplify}[10\,x3'[t] - x2[t] + x3[t]], \left\{\text{Cos}\left[\frac{\sqrt{3}\,t}{20}\right], \text{Sin}\left[\frac{\sqrt{3}\,t}{20}\right]\right\}\right] == 0$$

$$-b1 + c1 + \frac{1}{2}\left(-2\,b2 - c2 + \sqrt{3}\,c3\right)e^{-3t/20}\,\text{Cos}\left[\frac{\sqrt{3}\,t}{20}\right] + \frac{1}{2}\left(-2\,b3 - \sqrt{3}\,c2 - c3\right)e^{-3t/20}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{20}\right] == 0$$

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c1 = a1; b1 = a1;
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From above eq. it is obvious that a1=b1=c1. Now solving for rest of the coefficients.

$$\text{Solve}\left[\frac{1}{2}\left(-a2 + \sqrt{3}\,a3 - 2\,c2\right) == 0, c2\right]$$

$$\left\{\left\{c2 \to \frac{1}{2}\left(-a2 + \sqrt{3}\,a3\right)\right\}\right\}$$

$$c2 = \frac{1}{2}\left(-a2 + \sqrt{3}\,a3\right);$$

c2 in terms of a2 and a3.

$$\text{Solve}\left[-\sqrt{3}\,a2 - a3 - 2\,c3 == 0, c3\right]$$

$$\left\{\left\{c3 \to \frac{1}{2}\left(-\sqrt{3}\,a2 - a3\right)\right\}\right\}$$

$$c3 = \frac{1}{2}\left(-\sqrt{3}\,a2 - a3\right);$$

c3 in terms of a2 and a3.

$$\text{Solve}\left[-2\,a2 - b2 + \sqrt{3}\,b3 == 0, b3\right]$$

$$\left\{\left\{b3 \to \frac{2\,a2 + b2}{\sqrt{3}}\right\}\right\}$$