

Name. _____

Applied Linear Algebra 2270-1
Sample Midterm Exam 1 In-Class
Tuesday, 17 Feb 2004

Instructions: This in-class exam is 50 minutes. No tables, notes, books or calculators allowed.

- 3. (Inverse of a matrix)** An $n \times n$ matrix A is said to have an inverse B if $AB = BA = I$, where I is the $n \times n$ identity matrix. Prove these facts:
1. If B_1 and B_2 are inverses of A , then $B_1 = B_2$.
 2. The inverse of the identity I is I .
 3. The zero matrix has no inverse.
 4. In checking the inverse relation $AB = BA = I$, only one of $AB = I$ or $BA = I$ needs to be verified.
- 4. (Elementary Matrices)** Let A be a 3×3 matrix and \vec{b} a vector in \mathcal{R}^3 . Define $C = \text{aug}(A, \vec{b})$. Let matrix F be obtained from C by the following: (a) Swap rows 2 and 3; (b) Add -1 times row 3 to row 1; (c) Swap rows 1 and 2; (d) Multiply row 2 by -5 . Write a matrix multiplication formula for F in terms of C and explicit elementary matrices.

5. (RREF method)

Let a and b denote constants and consider the system of equations

$$\begin{pmatrix} 1 & a+b & a \\ 0 & 0 & a \\ 1 & a+b & 2a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ a \\ a \end{pmatrix}$$

- (1) Determine those values of a and b such that the system has a solution.
- (2) For each of the values in (1), solve the system.
- (3) For each of the solutions in (2), check the answer.