Name _____ **2270-1**

Introduction to Linear Algebra 2270-1
Midterm Exam 1 Fall 2003

Take-Home Exam Date: Friday, 19 September, 2003
Inclass Exam Date: Tuesday, 23 September, 2003

Instructions. The four take-home problems below are to be submitted at class time at the date marked above. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.
The in-class portion of the exam is the last 15 minutes of class, one problem, of a type similar to either problem 3 or problem 4. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. **(Elementary matrices)** Let C be the agumented matrix of a system $A\mathbf{x} = \mathbf{b}$. Verify that each of the operations `swap`, `mult` and `combo` acting on C produces a matrix answer F of the form $F = EC$, where E is a square *elementary matrix*, obtained from the identity matrix by suitable modification. In particular, define E and supply a general proof for each of the operations `swap`, `mult` and `combo`.

2. **(Inverse of a matrix)** An $n \times n$ matrix A is said to have an inverse B if $AB = BA = I$, where I is the $n \times n$ identity matrix. Prove these facts:

   1. If B_1 and B_2 are inverses of A, then $B_1 = B_2$.

   2. The inverse of the identity I is I.

   3. The zero matrix has no inverse.

   4. In checking the inverse relation $AB = BA = I$, only one of $AB = I$ or $BA = I$ needs to be verified. You may reference a theorem or supply your own proof.

3. **(Gaussian algorithm)** Solve for x, y, z in the 3×3 linear system

$$\begin{array}{rcrcrcr} 2x & + & 2ay & + & cz & = & b \\ 3x & + & ay & + & 2cz & = & 2b \\ 5x & + & 3ay & + & 3cz & = & 3b \end{array}$$

   using the Gaussian algorithm, for all constant values of a, b, c. Include all algorithm details and an answer check for each of the three separate cases. Sanity check: $a \neq 0$ is one case, with parametric solution $x = 3b/4 - 3ct_1/4$, $y = -b/(4a) + ct_1/(4a)$, $z = t_1$. The case $a = 0$ has subcases $c \neq 0$ and $c = 0$, for one of which you will report *no solution*.

4. **(Inverse algorithm)** Determine by `rref` methods the inverse matrix of

$$A = \begin{pmatrix} 1 & a & 0 \\ a & 0 & b \\ 0 & 1 & 1 \end{pmatrix}.$$

   Please state conditions on a, b for when the inverse exists. Show all hand details. Include an answer check, preferably done in `maple`.

Please attach this exam or a copy to the front of your submitted exam on the due date. Kindly staple the left upper corner and write your name on all pages.