Chapter 5 EXERCISES, E&P.

5.1-9, p284 Solve for c_1, c_2 given $y = c_1y_1 + c_2y_2$, $y_1 = e^{-x}$, $y_2 = xe^{-x}$, subject to

$$(1) \quad \begin{cases} y(0) = 2, \\ y'(0) = -1. \end{cases}$$

$$\begin{cases} y = c_1e^{-x} + c_2xe^{-x} \\ y' = -c_1e^{-x} + c_2(e^{-x} - xe^{-x}) \end{cases}$$

substitute y_1, y_2
product rule

$$\begin{cases} y(0) = c_1e^0 + c_2(0) \\ \quad = c_1 + (0)c_2 \\ y'(0) = -c_1e^0 + c_2(e^0 - 0) \\ \quad = -c_1 + c_2 \end{cases}$$

substitute $x=0$ into The above equations.

$$\begin{cases} c_1 + 0c_2 = 2, \\ -c_1 + c_2 = -1. \end{cases}$$

Rewrite (1) using The above relations.
Get 2×2 system for c_1, c_2.

$$\begin{cases} c_1 + 0c_2 = 2, \\ 0c_1 + c_2 = 1 \end{cases}$$

To eqtn 2 add eqtn 1.

$$\begin{cases} c_1 = 2 \\ c_2 = 1 \end{cases}$$

system solved for c_1, c_2.

$$\boxed{y = 2e^{-x} + xe^{-x}}$$

solution found.

Check:

$$\begin{aligned} y(0) &= 2e^0 + 0 \\ &= 2 \ \checkmark \\ y'(0) &= (-2e^{-x} + e^{-x} - xe^{-x})\big|_{x=0} \\ &= -2e^0 + e^0 - 0 \\ &= -1 \ \checkmark \end{aligned}$$

5.1-36, p285 Find The general solution of

$$2y'' + 3y' = 0$$

Apply The recipe

$2r^2 + 3r = 0$ characteristic eqtn

$(2r+3)r = 0$ factor it

$r = 0, r = -3/2$ Distinct roots, Case 1

$y = c_1e^{0x} + c_2e^{-3x/2}$ Recipe Case 1 applied

$\boxed{y = c_1 + c_2e^{-3x/2}}$ simplify

5.1-38, p285 Find The general solution of

$$4y'' + 8y' + 3y = 0.$$

Apply The recipe

$4r^2 + 8r + 3 = 0$ char. eqtn.

$r = \frac{-8}{2(4)} \pm \frac{1}{2(4)}\sqrt{8^2 - 4(4)(3)}$ Quadratic formula $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\quad = -1 \pm \frac{\sqrt{3}}{4}$

$\boxed{y = c_1e^{-x+\sqrt{3}x/4} + c_2e^{-x-\sqrt{3}x/4}}$ Recipe case 1 applied.

5.1-40, p285 Find The general solution of

$$9y'' - 12y' + 4y = 0.$$

$9r^2 - 12r + 4 = 0$ char. eqtn for The recipe

$(3r-2)(3r-2) = 0$ Factor by inverse FOIL

$r = 2/3, r = 2/3$ Double root, Case 2

$\boxed{y = c_1e^{2x/3} + c_2xe^{2x/3}}$ Case 2 recipe applied

5.2-15

5.2-14, p247

Find y, given a basis
$y_1 = e^x,\ y_2 = e^{2x},\ y_3 = e^{3x}$
for $y''' - 6y'' + 11y' - 6y = 0$ subject to
(1) $y(0) = 0,\ y'(0) = 0,\ y''(0) = 3.$

$$\begin{cases} y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} \\ y' = c_1 e^x + 2c_2 e^{2x} + 3c_3 e^{3x} \\ y'' = c_1 e^x + 4c_2 e^{2x} + 9c_3 e^{3x} \end{cases}$$

General Solution

$$\begin{cases} y(0) = c_1 + c_2 + c_3 \\ y'(0) = c_1 + 2c_2 + 3c_3 \\ y''(0) = c_1 + 4c_2 + 9c_3 \end{cases}$$

Substitute x=0 in The above equations

$$\begin{cases} c_1 + c_2 + c_3 = 0 \\ c_1 + 2c_2 + 3c_3 = 0 \\ c_1 + 4c_2 + 9c_3 = 3 \end{cases}$$

Insert relations above into equations (1).

$$\begin{cases} c_1 + 0c_2 + 0c_3 = 3/2 \\ 0c_1 + c_2 + 0c_3 = -3 \\ 0c_1 + 0c_2 + c_3 = 3/2 \end{cases}$$

RREF for The above nonhomogeneous 3x3 system. In your solution, show The steps, by attaching hand details or Maple Code.

$c_1 = 3/2,\ c_2 = -3,\ c_3 = 3/2$

$$\boxed{y = \frac{3}{2}e^x - 3e^{2x} + \frac{3}{2}e^{3x}}$$

Check

$y(0) = \frac{3}{2} - 3 + \frac{3}{2}$
$= 0$ ✓

$y'(0) = c_1 + 2c_2 + 3c_3$
$= 0$ ✓

$y''(0) = c_1 + 4c_2 + 9c_3$
$= \frac{3}{2} - 12 + \frac{27}{2}$
$= 3$ ✓

5.2-21, 5.2-23

Example

Given $y'' + y = 6x + 1$, $y_c = c_1\cos x + c_2 \sin x$
$y_p = 6x + 1$ and $y(0) = 2,\ y'(0) = -2,$
find $y(x)$.

$y = y_c + y_p$

Structure of solutions

$$\begin{cases} y = c_1\cos x + c_2 \sin x + 6x + 1 \\ y' = -c_1 \sin x + c_2 \cos x + 6 \end{cases}$$

Substitute for y_c, y_p

$$\begin{cases} y(0) = c_1\cos 0 + c_2 \sin 0 + 1 \\ \quad = c_1 + 0c_2 + 1 \\ y'(0) = -c_1 \sin 0 + c_2 \cos 0 + 6 \\ \quad = 0c_1 + c_2 + 6 \end{cases}$$

$$\begin{cases} c_1 + 0c_2 + 1 = 2 \\ 0c_1 + c_2 + 6 = -2 \end{cases}$$

substitute The above relations into $y(0)=2$, $y'(0)=-2$

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -8 \end{array}\right)$$

augmented matrix. Already in rref.

$c_1 = 1,\ c_2 = -8$

$$\boxed{y = \cos x - 8\sin x + 6x + 1}$$

Solution found

Check

$y(0) = \cos 0 - 8\sin 0 + 1$
$= 1 + 0 + 1$
$= 2$ ✓

$y'(0) = -\sin 0 - 8\cos 0 + 6$
$= 0 - 8 + 6$
$= -2$ ✓

P310, 5.3-4 Find the general solution of $2y''-7y'+3y=0$

$2r^2-7r+3=0$	characteristic equation for the recipe
$(2r-1)(r-3)=0$	
$r=1/2\ ,\ r=3$	roots found
$y=c_1e^{x/2}+c_2e^{3x}$	Recipe case 1 solution

5.3-5

Example. solve $y''+4y'+4y=0.$

$r^2+4r+4=0$	char. eqtn.
$(r+2)^2=0$	factor
$r=-2,\ r=-2$	Double root, Case 2 of recipe
$y=c_1e^{-2x}+c_2xe^{-2x}$	Recipe applied

5.3-8, 5.3-9

Example. Solve $y''+8y'+27y=0.$

$r^2+8r+27=0$	char. eqtn.
$(r+4)^2+11=0$	Complete the square
$r+4=\pm\sqrt{11}\,i$	Solve by roots
$Re(r)=-4$ $Im(r)=\sqrt{11}$	get real and imaginary parts of roots for case 3
$y=c_1e^{-4x}\cos\sqrt{11}x+c_2e^{-4x}\sin\sqrt{11}x$	Case 3 applied

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5.3-21

Example Solve $\begin{cases} y''-6y'+21y=0, \\ y(0)=3,\ \ y'(0)=1 \end{cases}$

$r^2-6r+21=0$	Char. eqtn.
$(r-3)^2+12=0$	Complete the square
$r-3=\pm\sqrt{12}\,i$	Solve by roots
$Re(r)=3$ $Im(r)=2\sqrt{3}$	Get the real and imaginary parts of the roots for case 3
$y=c_1e^{3x}\cos 2\sqrt{3}x+c_2e^{3x}\sin 2\sqrt{3}x$	Case 3 applied.
$y'=3\left[c_1e^{3x}\cos 2\sqrt{3}x+c_2e^{3x}\sin 2\sqrt{3}x\right]+2\sqrt{3}\left[-c_1e^{3x}\sin 2\sqrt{3}x+c_2e^{3x}\cos 2\sqrt{3}x\right]$	find y'
$\begin{cases} y(0)=c_1+0c_2 \\ y'(0)=3c_1+2\sqrt{3}c_2 \end{cases}$	Substitute $x=0$ into preceding equations
$\begin{cases} c_1+0c_2=3 \\ 3c_1+2\sqrt{3}c_2=1 \end{cases}$	Substitute $y(0)=3$, $y'(0)=1$
$\left[\begin{array}{cc\|c} 1 & 0 & 3 \\ 3 & 2\sqrt{3} & 1 \end{array}\right]$	augmented matrix
$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -4/\sqrt{3} \end{bmatrix}$	rref
$c_1=3,\ c_2=-4/\sqrt{3}$	Solve for c_1, c_2.
$\boxed{y=3e^{3x}\cos 2\sqrt{3}x-\frac{4}{\sqrt{3}}e^{3x}\sin 2\sqrt{3}x}$	Solution found

Check: $y(0)=3e^0\cos 0-\frac{4}{\sqrt{3}}e^0\sin 0$
$=3$ ✓

$y'(0)=\left[9\cos 2\sqrt{3}x-6\sqrt{3}\sin 2\sqrt{3}x-\frac{12}{\sqrt{3}}\sin 2\sqrt{3}x-8\cos 2\sqrt{3}x\right]e^{3x}\Big|_{x=}$
$=9-0+0-8$
$=1$ ✓

5.3 - 30. Solve $y^{iv} - y''' + y'' - 3y' - 6y = 0$.

$r^4 - r^3 + r^2 - 3r - 6 = 0$	• Char eqtn.
$r = -1$ is a root	• Tried possible rational roots $\pm 1, \pm 2, \pm 3, \pm 6$
$r+1 \overline{) r^4 - r^3 + r^2 - 3r - 6}$ with quotient $r^3 - 2r^2 + 3r - 6$, remainder 0	• Long division
$(r+1)(r^3 - 2r^2 + 3r - 6) = 0$	• Factoral form
$r^3 - 2r^2 + 3r - 6 = 0$	• Examine roots of Cubic
$r = 2$ is a root	• tried $\pm 1, \pm 2, \pm 3, \pm 6$
$(r-2)(r^2 + 3) = 0$	• Long division, factored.
$(r+1)(r-2)(r^2+3) = 0$	• Factored form of the original quartic.
$r = -1, 2, \pm\sqrt{3}i$	• roots found
$y =$ gen sol of $(D+1)y = 0$ $+$ gen sol of $(D-2)y = 0$ $+$ gen sol of $(D^2+3)y = 0$	• Solve via distinct factors
$y = c_1 e^{-x}$ $+ c_2 e^{2x}$ $+ c_3 \cos\sqrt{3}x + c_4 \sin\sqrt{3}x$	• Recipe applied for each case.

5.4-3, 5.3-4

Example. A mass of 6 kg is attached to a spring. This spring stretches 25 cm due to a 14 Newton force. At $t=0$, $x(0) = 0$, $x'(0) = -10$. Find a formula for $x(t)$, the amplitude A, period T and frequency f.

$mx'' + \beta x' + kx = 0$	Spring-mass equation
$6x'' + \frac{14}{1/4}x = 0$	$m = 6$, $\beta = 0$, $k =$ force/ $= 14/\frac{1}{4}$.
$x'' + \frac{28}{3}x = 0$	
$r^2 + \frac{28}{3} = 0$	Char. eqtn.
$r = \pm 2\sqrt{7/3}\, i$	Roots found
$x = c_1\cos\omega t + c_2\sin\omega t$	$\omega = 2\sqrt{7/3}$
→ $x = -\frac{10}{\omega}\sin(\omega t)$	Because $x(0) = 0$, $x'(0) =$
→ period $= \frac{2\pi}{\omega}$ $= \pi/\sqrt{7/3}$	
→ Frequency $= 1/\text{period}$ $= \sqrt{7/3}/\pi$	
→ Amplitude $= \sqrt{c_1^2 + c_2^2}$ $= \frac{10}{\omega}$	

5.4-17,
5.4-19

Find $x(t)$ and classify as Overdamped, Critically damped or underdamped.

<u>5.4-18</u>

$$m x'' + \beta x' + k x = 0,$$
$$x(0) = 0, \quad x'(0) = -8,$$
$$m = 2, \quad \beta = 12, \quad k = 50$$

$2r^2 + 12r + 50 = 0$ — Characteristic equation

$r = -3 \pm 4i$ — roots, quadratic formula

→ $x(t) = c_1 e^{-3t} \cos 4t + c_2 e^{-3t} \sin 4t$ — Recipe case 3 applied

$x' = -3x + e^{-3t}[-4c_1 \sin 4t + 4c_2 \cos 4t]$

$$\begin{cases} x(0) = c_1 + 0c_2 \\ x'(0) = -3c_1 + 4c_2 \end{cases}$$

set $t = 0$ in relations for x and x',

$$\begin{cases} c_1 + 0c_2 = 0, \\ -3c_1 + 4c_2 = -8. \end{cases}$$

Substitute $x(0)=0$, $x'(0) = -8$

$c_1 = 0, \quad c_2 = -2$ — solve by back-subst.

$\boxed{x(t) = -2e^{-3t} \sin(4t)}$ — solution to The IVP.

Check

$x(0) = -2e^0 \sin 0$
$= 0,$

$x'(0) = 6e^0 \sin 0 - 8e^0 \cos 0$
$= -8$

Classification

$\boxed{\text{underdamped}}$

5.5-5, 5.5-4
Example. Solve for y_p, $y'' + y' + 3y = \cos^2 x$.

$y'' + y' + 3y = \cos^2 x$
$= \frac{1}{2} + \frac{1}{2} \cos 2x$

Trig identity a
$\cos 2\theta = 2\cos$

Easy-to-solve problems, $y_p = y_1 + y_2$

① $y_1'' + y_1' + 3y_1 = \frac{1}{2}$

② $y_2'' + y_2' + 3y_2 = \frac{1}{2} \cos(2x)$

- To solve ① use The equilibrium method, getting

$$y_1 = \frac{1}{6}$$

- To solve ② use Kümmer's method (book meth
The same s

$$\begin{cases} y_2 = \operatorname{Re}(\Psi e^{2ix}) \\ [(D+2i)^2 + (D+2i) + 3]\Psi = \frac{1}{2} \end{cases}$$

Then
$$[D^2 + (4i+1)D + (2i-1)]\Psi = \frac{1}{2}$$
has by The equilibrium method solution
$$\Psi = \frac{1}{2}\,\frac{1}{2i-1}$$
$$= -\frac{1}{10}(1+2i)$$

Then

$y_2 = \operatorname{Re}(\Psi e^{2ix})$
$= -\frac{1}{10} \operatorname{Re}(1+2i\,(\cos 2x + i \sin 2x))$
$= -\frac{1}{10}(\cos 2x - 2 \sin 2x)$

$y_p = y_1 + y_2$
$= \boxed{\frac{1}{6} - \frac{1}{10} \cos 2x + \frac{1}{5} \sin 2x}$

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5.5-11 P337

5.5-12 Solve $y''' + y' = 2 - \sin x$ for y_p

Easy-To-Solve problems

① $y_1''' + y_1' = 2$

② $y_2''' + y_2' = -\sin x$

$y = y_1 + y_2$

Solve ① by the equilibrium method

$y_1' = 2$

$\boxed{y_1 = 2x}$

Equil Eqtn, extended Quadrature. No constants to evaluate!

Solve ② by Kümmer's method (book method slower)

$y_2 = \mathrm{Im}(\Upsilon e^{ix})$

$[(D+i)^3 + (D+i)]\Upsilon = -1$

$[D^3 + 3i D^2 - 2D]\Upsilon = -1$ Expanded

$-2\Upsilon' = -1$ Equil Eqtn

→ $\Upsilon = \frac{1}{2}x$ Quadrature. No constants to find.

$y_2 = \mathrm{Im}(\frac{1}{2}x e^{ix})$

$\boxed{y_2 = \frac{1}{2}x \sin x}$

$y = y_1 + y_2$

$\boxed{y = 2x + \frac{x}{2}\sin x}$

Check: By hand (very long).

5.5-19

5.5-18 P 327 Solve for y_p: $y^{iv} - 5y'' + 4y = e^x - xe^{2x}$

Easy-to-solve problems

① $y_1^{iv} - 5y_1'' + 4y_1 = e^x$

② $y_2^{iv} - 5y_2'' + 4y_2 = -xe^{2x}$

$y_p = y_1 + y_2$

Answer

$y = y_1 + y_2$

$= -\frac{x}{6}e^x + (\frac{19}{144}x - \frac{x^2}{24})e^{2x}$

Details below.

Checked by hand (very long)

① by Kummer's method

$$\begin{cases} y_1 = \Upsilon e^x \\ [(D+1)^4 - 5(D+1)^2 + 4]\Upsilon = 1 \end{cases}$$

$[D^4 + 3D^3 + D^2 - 6D]\Upsilon = 1$ Expand

$-6\Upsilon' = 1$ Equil extended method

$\Upsilon = -x/6$ Quadrature. No equatio to solve for constants!

$\boxed{y_1 = -\frac{x}{6}e^x}$

② by Kümmer's Method

$$\begin{cases} y_2 = \Upsilon e^{2x} \\ [(D+2)^4 - 5(D+2)^2 + 4]\Upsilon = -x \end{cases}$$

$$(1) \begin{cases} [D^4 + 8D^3 + 19D^2 + 12D]\Upsilon = -x \\ [D^5 + 8D^4 + 19D^3 + 12D^2]\Upsilon = -1 \end{cases}$$ Expanded DE Diff until RHS = C

$12\Upsilon'' = -1$ Extended equil meth

$\Upsilon = c_0 + c_1 x + c_2 \frac{x^2}{2}$ Form of the solution

$$(2) \begin{cases} 0 + 0 + 19c_2 + 12c_1 = 0 \\ 0 + 0 + 0 + 12c_2 = -1 \end{cases}$$ Set $x = 0$ in (1) to get

$\Upsilon = \frac{19}{144}x - \frac{x^2}{24}$ Solve $c_0 \equiv 0, c_1 = \frac{19}{144}, c_2 =$

$\boxed{y_2 = [\frac{19}{144}x - \frac{x^2}{24}]e^{2x}}$ Used $y_2 = \Upsilon e^{2x}$

5.5-35
5.5-33
Example.

Solve $\begin{cases} y''+y = x+\sin 2x \\ y(0)=0,\ y'(0)=1/3 \end{cases}$

Easy-to-solve problems

$$\begin{cases} y_p = y_1+y_2 \\ y_1''+y_1 = x \\ y_2''+y_2 = \sin 2x \end{cases}$$

To solve $y_1''+y_1 = x$

(1) $\begin{cases} y_1''+y_1 = x \\ y_1'''+y_1' = 1 \end{cases}$ Differentiate until the RHS is constant

$y_1' = 1$ Equilibrium extended method

$y_1 = a+bx$ Quadrature

(2) $\begin{cases} 0+a = 0 \\ 0+b = 1 \end{cases}$ substitute $x=0$ in (1) to get (2), using $y_1 = a+bx$

$\boxed{y_1 = x}$ Insert $a=0, b=1$.

To solve $y_2''+y_2 = \sin 2x$

$\begin{cases} y_2 = \operatorname{Im}(\mathcal{Z} e^{2ix}) \\ [(D+2i)^2+1]\mathcal{Z} = 1 \end{cases}$ Kummer's method applies

$[D^2+4iD-3]\mathcal{Z} = 1$ Expand DE

$-3\mathcal{Z} = 1$ Equilibrium method applied

$y_2 = \operatorname{Im}\left(\frac{-1}{3}e^{2ix}\right)$

$= \boxed{\frac{-1}{3}\sin 2x}$

$y_p = y_1+y_2$

$= \boxed{x - \frac{1}{3}\sin 2x}$

Found y_p. satisfies $y(0)=0$, $y'(0)=1/3$. So $y = y_p$!

5.5-47, 5.5-49,
5.5-51,
5.5-48 P 337

Solve by variation of parameters
$y''-2y'-8y = 3e^{-2x}$

Answer: $y_p = \frac{1}{12}e^{4x} - \frac{1}{12}e^{-2x} - \frac{x}{2}e^{-2x}$
$y_h = c_1e^{4x}+c_2e^{-2x}$
Collected terms give
$y = c_1e^{4x}+c_2e^{-2x} - \frac{x}{2}e^{-2x}$ absorbed into arb constants

Details

$r^2-2r-8=0$ char eqtn
$(r-4)(r+2)=0$ Factored
$y_h = c_1e^{4x}+c_2e^{-2x}$ Recipe case

Cauchy Kernel

$$K(x,t) = \frac{\begin{vmatrix} e^{4t} & e^{-2t} \\ e^{4x} & e^{-2x} \end{vmatrix}}{\begin{vmatrix} e^{4t} & e^{-2t} \\ 4e^{4t} & -2e^{-2t} \end{vmatrix}}$$
$$= \frac{1}{6}e^{4x-4t} - \frac{1}{6}e^{2t-2x}$$

Calculate y_p by var of param

$y_p(x) = \int_0^x K(x,t)(3e^{-2t})\,dt$ $y = \int_{x_0}^x K f\,dt$
$= \frac{3}{6}\int_0^x (e^{4x-4t-2t} - e^{2t-2x-2t})\,dt$ Subst for K
$= -\frac{1}{12}e^{-2x} + \frac{1}{12}e^{4x} - \frac{x}{2}e^{-2x}$ Evaluate int

Checked by hand (7 lines)

5.6-3, P 348

Example Solve $x''+16x = 5\cos 5t + 10\sin 5t$, $x(0)=1$, $x'(0)=0$. Write x as the sum x_1+x_2 where the frequencies of x_1, x_2 match resp. the frequencies of $x''+16x=0$ and the forcing term.

Find x_1

$$x_1''+16x_1=0$$
$$r^2+16=0$$
$$r=\pm 4i$$
$$\boxed{x_1 = c_1\cos 4t + c_2 \sin 4t}$$

answer:

$$x = \frac{14}{9}\cos 4t + \frac{25}{18}\sin 4t + \left(\frac{-5}{9}\right)\cos 5t + \left(\frac{-10}{9}\right)\sin 5t$$

Find x_2

$$\boxed{x_2 = -\frac{1}{9}(5\cos 5t + 10\sin 5t)}$$

This was found by classical undetermined coefficients using trial solution

$$x = a\cos 5t + b\sin 5t.$$

$$\begin{cases}(16-25)a = 5\\(16-25)b = 10\end{cases}$$
$$a=-\frac{5}{9},\quad b=\frac{-10}{9}$$

Find c_1, c_2

$$\begin{cases}c_1 - \frac{5}{9} = 1\\4c_2 + 5\left(\frac{-10}{9}\right)=0\end{cases}$$

Substitute $x = x_1+x_2$ into $x(0)=1$, $x'(0)=0$.

$$c_1 = \frac{14}{9},\quad c_2 = \frac{25}{18}$$

Check

$$x(0) = x_h(0)+x_p(0) = \frac{14}{9}-\frac{5}{9} = 1 \checkmark$$

$$x'(0) = \frac{25}{18}(4) - \frac{50}{9} = 0 \checkmark$$

5.6-9, 5.6-11

5.6-8

Find the steady-state periodic solution.

$$x''+3x'+5x = -4\cos(5t)$$

answer: $x_{ss} = \frac{4}{125}(4\cos 5t - 3\sin 5t)$

= Terms in the general solution not involving negative exponentials

Homogeneous Solution x_h

$$x''+3x'+5x=0$$
$$r^2+3r+5=0$$
$$r = -\frac{3}{2} \pm \frac{\sqrt{11}}{2} i \qquad \text{Quadratic formula}$$
$$x_h = \left(c_1\cos\frac{\sqrt{11}\,t}{2} + c_2\sin\frac{\sqrt{11}}{2}t\right)e^{-3t/2} \qquad \text{Recipe Case 3}$$

Particular Solution x_p

$$\begin{cases}x_p = \operatorname{Re}(Y e^{5it})\\ \left[(D+5i)^2+3(D+5i)+5\right]Y = -4\end{cases} \qquad \text{Kummer's Transform}$$

$$\left[D^2 + (10i+3)D + 15i - 20\right]Y = -4 \qquad \text{Expand}$$

$$Y = \frac{-4}{15i-20} \qquad \text{Equil. method applied}$$

$$x_p = \operatorname{Re}(Y e^{5it})$$
$$= \frac{4}{125}\operatorname{Re}\left((3i+4)e^{5it}\right)$$
$$= \frac{4}{125}(4\cos 5t - 3\sin 5t)$$

$$\frac{4}{20-15i} = \frac{4(20+15i)}{20^2+15^2} = \frac{4}{25}(20+15i)\cdot\frac{1}{25} = \frac{4}{125}(4+3i)$$

Checked by hand.

Steady-state

Since $x = x_h + x_p$ and x_h involves negative exponentials, then x_h is the transient and x_p is the steady-state.