problem. Solve \( y'' + 3y' + 2y = e^{-x} + x \) by the method of undetermined coefficients.

solution: \( y = y_h + y_p \) (Edwards-Penney p149)

\[ y_h = \text{solution of } y'' + 3y' + 2y = 0 \text{ part I} \]

obtained from the "recipe" (Edwards-Penney 5.2)

\[ y_p = \text{trial solution in the method of undetermined coefficients with coefficients evaluated} \text{ part II} \]

(Edwards-Penney p331)

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**Part I** Find \( y_h \)

\[ r^2 + 3r + 2 = 0 \]

\[ (r+2)(r+1) = 0 \]

\[ r = -2, r = -1 \]

\[ y_h = c_1 e^{-x} + c_2 e^{-2x} \]

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**Part II** Find \( y_p \)

**Determine Trial Solution**

\[ \text{Trial}_1 = k_1 + k_2 x \]

\[ \text{Trial}_2 = c e^{-x} \]

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**A particular solution of the nonhomogeneous DE**.

**Recipe solution** see p138 E-P.

**The solution of the homogeneous DE**.

**Characteristic equation for**

\[ y'' + 3y' + 2y = 0 \]

**Recipe solution** see p138 E-P.

**Trial solution for**

\[ f_1(x) = x \text{, see p331 E-P.} \]

**Trial solution for**

\[ f_2(x) = e^{-x} \]
Trial_1 = k_1 + k_2x \\
     (not modified)

Trial_2 = Cx e^{-x} \\
     (modified)

Trial = Trial_1 + Trial_2
     = k_1 + k_2x + Cx e^{-x}

Determine coefficients k_1, k_2, C

\[ y = k_1 + k_2x + Cx e^{-x} \]
\[ y' = 0 + k_2 + C e^{-x} - Cx e^{-x} \]
\[ y'' = 0 + 0 - 2C e^{-x} + Cx e^{-x} \]

\[ y'' + 3y' + 2y = e^{-x} + x \]
\[ (-2Ce^{-x} + Cxe^{-x}) \]
\[ + 3(k_2 + Ce^{-x} - Cxe^{-x}) \]
\[ + 2(k_1 + k_2x + Cxe^{-x}) \]
\[ = e^{-x} + x \]

Fix up rule 2 \hspace{1cm} \text{p.331}

Trial_1 not of \text{Re form} \\
y = x = Ce^{-x} + C_2 e^{-2x} \\

Fix up rule 2 \hspace{1cm} \text{p.331}

Trial_2 = x = Ce^{-x} \text{ was of} \\
\text{Re form} \hspace{0.5cm} C_1 e^{-x} + C_2 e^{-2x}, \\
\text{so mult by} x.

Sum Rule (13) \hspace{1cm} \text{p.330}

Let \[ y = \text{trial solution} \] \\
in the DE: \\
y'' + 3y' + 2y = e^{-x} + x \\
and find k_1, k_2, C.

Find y, y', y'' formulas.

Given DE

Substitute formulas into the DE \\
y'' + 3y' + 2y = e^{-x} + x
undetermined coefficients

\[( -2c + 3c)e^{-x} + (c - 3c + 2c)xe^{-x} + (3k_2 + 2k_1) + 2k_2x = e^{-x} + x \]

\[
\begin{cases}
-2c + 3c = 1 \\
c - 3c + 2c = 0 \\
3k_2 + 2k_1 = 0 \\
2k_2 = 1
\end{cases}
\]

\[
k_2 = \frac{1}{2} \]

\[
k_1 = -\frac{3}{2}k_2 = -\frac{3}{2} \cdot \frac{1}{2} = -\frac{3}{4}
\]

\[c = 1\]

**Substitute answers into trial solution**

**Trial** = \[k_1 + k_2x + cxe^{-x}\]

\[y_p = -\frac{3}{4} + \frac{1}{2}x + xe^{-x}\]

**Apply General Solution Theory**

**Final Answer**

\[y = y_h + y_p \rightarrow y = c_1e^{-x} + c_2e^{-2x} - \frac{3}{4} + \frac{1}{2}x + xe^{-x}\]

**Rearrange Terms**

Determine equations for \(k_1, k_2, c\) by matching terms on LHS and RHS

solve for \(k_2\)

solve for \(k_1\)

solve for \(c\)

**Define \(y_p\)**

\[y = y_h + y_p\]

This answer was checked on scratch paper to be correct.