problem. Solve $y'' + 3y' + 2y = e^{-x} + x$
by The method of undetermined coefficients.

Solution: $y = y_h + y_p$ (Edwards-Penney p149)

y_h = Solution of $y'' + 3y' + 2y = 0$ obtained from The "Recipe" (Edwards-penney 5.2) — part I below

y_p = trial solution in The method of undetermined coefficients with coefficients evaluated (Edwards-penney p 331) — part II below

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Part I Find y_h	The Solution of The homogeneous DE.
$r^2 + 3r + 2 = 0$ $(r+2)(r+1) = 0$ $r = -2,\ r = -1$	Characteristic equation for $y'' + 3y' + 2y = 0$
$y_h = c_1 e^{-x} + c_2 e^{-2x}$	Recipe solution see p 138 E-P
Part II Find y_p. $\boxed{\text{Determine Trial Solution}}$	A particular solution of The nonhomogeneous DE.
$\text{Trial}_1 = k_1 + k_2 x$	Trial Solution for $f_1(x) = x$. See p 331 E-P.
$\text{Trial}_2 = C e^{-x}$	trial solution for $f_2(x) = e^{-x}$

$\text{Trial}_1 = k_1 + k_2 x$
(not modified)

Fixup rule 2 P 331
Trial_1 not of the form
$y_h = c_1 e^{-x} + c_2 e^{-2x}$

$\text{Trial}_2 = c x e^{-x}$
(modified)

Fixup rule 2 P 331
$\text{Trial}_2 = c e^{-x}$ was of the form $c_1 e^{-x} + c_2 e^{-2x}$,
so mult by x.

$\text{Trial} = \text{Trial}_1 + \text{Trial}_2$
$= k_1 + k_2 x + c x e^{-x}$

Sum rule (13) p 330

$\boxed{\text{Determine coefficients } k_1, k_2, c}$

Let y = trial solution in the DE:
$y'' + 3y' + 2y = e^{-x} + x$
and find k_1, k_2, c.

$y = k_1 + k_2 x + c x e^{-x}$
$y' = 0 + k_2 + c e^{-x} - c x e^{-x}$
$y'' = 0 + 0 - 2c e^{-x} + c x e^{-x}$

Find y, y', y'' formulas.

$y'' + 3y' + 2y = e^{-x} + x$

Given DE

$1(-2c e^{-x} + c x e^{-x})$
$+3(k_2 + c e^{-x} - c x e^{-x})$
$+2(k_1 + k_2 x + c x e^{-x})$
$= e^{-x} + x$

substitute formulas into the DE
$y'' + 3y' + 2y' = e^{-x} + x$

$$(-2C+3C)e^{-x}+(C-3C+2C)xe^{-x}$$
$$+(3K_2+2K_1)+2K_2x=e^{-x}+x$$

Rearrange Terms

$$\begin{cases} -2C+3C=1 \\ C-3C+2C=0 \\ 3K_2+2K_1=0 \\ 2K_2=1 \end{cases}$$

Determine equations for K_1, K_2, C by matching Terms on LHS and RHS

$$K_2=\frac{1}{2}$$

Solve for K_2

$$K_1=-\frac{3}{2}K_2$$
$$=-\frac{3}{2}\frac{1}{2}$$
$$=-3/4$$

Solve for K_1

$$C=1$$

Solve for C

Substitute answers into Trial Solution

$$\text{Trial}=K_1+K_2x+Cxe^{-x}$$
$$y_p=-\frac{3}{4}+\frac{1}{2}x+xe^{-x}$$

Define y_p

Apply General Solution Theory

$y=y_h+y_p$

$$y=y_h+y_p$$

Final answer
$$\longrightarrow y=c_1e^{-x}+c_2e^{-2x}-\frac{3}{4}+\frac{1}{2}x+xe^{-x}$$

This answer was checked on scratch paper to be correct.