P53 # 3. $y' + 3y = 2x e^{-3x}$. Solve by the factorization method.

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$y' + (3)y = 2x e^{-3x}$ — standard form $y' + py = q$

$P = \int (3)\,dx$ — primitive $P = \int p\,dx$

$= 3x$ — simplify constants

$e^P = e^{3x}$ — simplified $e^P$

Find Quadrature form

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$y' + (3)y = 2x e^{-3x}$ — std form

$\dfrac{(e^P y)'}{e^P} = 2x e^{-3x}$ — Replace $y' + py$ by $\dfrac{(e^P y)'}{e^P}$

$\dfrac{(e^{3x} y)'}{e^{3x}} = 2x e^{-3x}$ — substitute $e^P = e^{3x}$

$(e^{3x} y)' = 2x$ — use $e^{3x} e^{-3x} = e^0 = 1$ after cross-multiply.

Apply method of Quadrature

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$\int (e^{3x} y)'\,dx = \int 2x\,dx$ — Apply quadrature to the quadrature form above

$e^{3x} y = x^2 + c$ — Fund. Thm. of calculus

$y = (x^2 + c)e^{-3x}$ — Divide

Report ans and check

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$\boxed{y = (x^2 + c)e^{-3x}}$

ans checks with textbook

P 53 #5 $xy' + 2y = 3x$, $y(1) = 5$. Solve by the factorization method.

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$y' + \left(\frac{2}{x}\right)y = 3$ — Standard form $y' + py =$

$P = \int \left(\frac{2}{x}\right)dx$ — primitive $P = \int p\,dx$

$= 2\ln x$

$= \ln x^2$

$e^P = e^{\ln x^2}$

$= x^2$ — simplified $e^P$ found

Find Quadrature Form

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$y' + \left(\frac{2}{x}\right)y = 3$ — std form copied

$\dfrac{(e^P y)'}{e^P} = 3$ — Replace $y' + py$ by $\dfrac{(e^P y)'}{e^P}$

$\dfrac{(x^2 y)'}{x^2} = 3$ — Substitute $x^2$ for $e^P$

$(x^2 y)' = 3x^2$ — Quadrature form found

method of Quadrature

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$\int (x^2 y)'\,dx = \int 3x^2\,dx$ — Apply quadrature to the previous line.

$x^2 y = x^3 + c$

$y = x + c/x^2$ — Divide. Solution Candidate found.

Report answer and check

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$5 = 1 + \frac{c}{1^2}$ — Substitute $x=1$, $y=5$ to find $c=4$.

answer $\boxed{y = x + \frac{4}{x^2}}$

answer checks with text.

p53 #11  $xy' + y = 3xy$, $y(1)=0$  Solve for $y(x)$ by the factorization method.

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$y' + (\frac{1}{x})y = 3y$  Divide by $x$

$y' + (\frac{1}{x} - 3)y = 0$  Standard form $y' + py = q$

$P = \int (\frac{1}{x} - 3)\,dx$  primitive $P = \int p\,dx$

$\quad = \ln x - 3x$

$e^P = e^{\ln x} e^{-3x}$

$\quad = x e^{-3x}$  simplified $e^P$

Find Quadrature Form

$y' + (\frac{1}{x} - 3)y = 0$  Copy of std form

$\frac{(e^P y)'}{e^P} = 0$  Replace $y' + py$ by $\frac{(e^P y)'}{e^P}$

$(e^P y)' = 0$  cross-multiply

$(x e^{-3x} y)' = 0$  Substitute for $e^P$. Quadrature form found.

Method of Quadrature

$\int (x e^{-3x} y)' dx = \int 0\, dx$  Method of quadrature applied

$x e^{-3x} y = c$

$y = \frac{c}{x} e^{3x}$  Divide. Candidate Solution found.

Report answer and Check

$0 = \frac{c}{1} e^{3}$  Substitute $x=1$, $y=0$ [from $y(1)=0$] to find $c = 0$.

$\boxed{y = 0}$

ans checks with book.

p53 #33. A tank contains 1000 liters of brine, 100 ... is salt. Pure water enters at 5 liters/sec ... uniformly mixed brine exits at 5 liters/s ... Find the time $t$ when the amount $x(t)$ of salt equals 10 kg.

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Model

Apply $\frac{dx}{dt} = r_i c_i - \frac{r_o}{V} x$ where $r_o = r_i = 5$, $c_i = 0$, $V = 1000$ and $x(0) = 100$. Then

$$\begin{cases} \frac{dx}{dt} = 0 - \frac{5}{1000} x \\ x(0) = 100 \end{cases}$$

is the model.

Solve the model.

This is a growth-decay model. The solution is

$x(t) = x(0) e^{-5t/1000}$

$\quad = 100 e^{-t/200}$

Find time t.

$10 = 100 e^{-t/200}$  Need $x(t) = 10$

$0.1 = e^{-t/200}$

$\ln 0.1 = -t/200$

$t = 200 \ln 10$  $0.1 = \frac{1}{10}$ and $\ln \frac{1}{10} = -\ln 10$.

$\quad = 461$ seconds

Report answer and Check

461 seconds

Ans checks with book (7 min 41 sec)