Tyson Black

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Memorize!

$y' = ky$

Growth-Decay Equation

Solution $y = y_0 e^{kx}$

y_0 = an arbitrary constant $= y(0)$

$\frac{du}{dt} = -h(u - u_0)$

Newton's Cooling Equation

Solution $u = u_0 + A_0 e^{-ht}$

Obtained by changing $y = u - u_0$ to get $y' = -hy$, Then apply the recipe above.

$\frac{dP}{dt} = (a - bP)P$

Verhulst Logistic Equation

Solution

$$p(t) = \frac{aP_0}{bP_0 + (a - bP_0)e^{-at}}$$

where $P_0 = P(0)$ = initial population.

obtained by changing

$$y = \frac{P}{a - bP}$$

to get $y' = ay$, Then apply the recipe above.

P 17 ① Find a function $y = f(x)$ satisfying the given differential equation and the prescribed initial condition.

$\frac{dy}{dx} = 2x + 1$; $y(0) = 3$	Given
Steps Skipped → $y(x) = \int (2x+1)\,dx$	integration of both sides of equ.
$y(x) = x^2 + x + C$	F.T.C. applied + Tables
$y(0) = 0 + 0 + C$	use of $y(0) = 3$
$0 + 0 + C = 3$ $C = 3$	
$y(x) = x^2 + x + 3$	Substitute 3 for C

Check:

Back of Book

Fundamental Theorem of Calculus

(a) $\int_a^b f'(x)dx = f(b) - f(a)$

(b) $\left(\int_a^x g(t)dt\right)' = g(x)$

Isaac Newton found these formulas in an effort to extend the formula D=RT to the case of instantaneous rates.

The method of Quadrature

- Applies to equations like $y' = 2x$
- Uses the fundamental theorem of calculus
- only produces a candidate solution — it does not verify the equation.

Example Solve by the method of quadrature $y'' = 2x$

Solution:

$\int y'dx = \int 2x dx$	Integrate both sides on x.
$y(x) + C_1 = x^2 + C_2$	Apply fund. Thm. Calc.
$y(x) = x^2 + C$	collect constants. Candidate found.

Apply the method of quadrature to solve

$$\begin{cases} \frac{dy}{dx} = 2x+1, \\ y(0) = 3. \end{cases}$$

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$y' = 2x+1$	Given DE
$\int y'dx = \int (2x+1)dx$	Integrate across both sides on x. (Multiply by dx, draw $\int$ sign ac
$y = x^2 + x + c$	Fund. Thm. of calculu applied; c = consta
$3 = 0^2 + 0 + c$	use $y=3$ at $x=0$ (given as $y(0)=3$)
$c = 3$	
$y = x^2 + x + 3$	Candidate solution found.

Check:

$LHS = y' = (x^2+x+3)' = 2x+1 = RHS$	LHS = left hand side of $y' = 2x+1$, RHS = right hand side. DE verified
$y(0) = 0^2+0+3 = 3$	Initial condition $y(0)=$ is verified.
$y = x^2 + x + 3$	Solution.

Find a function $y = y(x)$ which satisfies the differential equation $\frac{dy}{dx} = (x-2)^2$ and initial condition $y(2) = 1$.

$y'(x) = (x-2)^2$	Given
$y'(x)\,dx = (x-2)^2\,dx$	Apply the method of quadrature
$\int y'(x)\,dx = \int (x-2)^2\,dx$	$\downarrow$
$y(x) = \frac{(x-2)^3}{3} + C$	
$1 = \frac{(2-2)^3}{3} + C$	Use $y(2)=1$ to find C
$C = 1$	
$y(x) = \frac{(x-2)^3}{3} + 1$	Candidate solution

Check:

$LHS = y'(x)$	Left side of DE
$= \left[\frac{(x-2)^3}{3} + 1\right]'$	
$= (x-2)^2 + 0$	
$= RHS$	DE verified
$LHS = y(2)$	Left side of IC
$= \left[\frac{(x-2)^3}{3} + 1\right]\Big\|_{x=2}$	
$= 0 + 1$	
$= RHS$	

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Solve $\frac{dy}{dx} = \frac{1}{\sqrt{x+2}}$, $y(2) = -1$.

$y'(x) = \frac{1}{\sqrt{x+2}}$	Given DE
$\int y'(x)\,dx = \int \frac{dx}{\sqrt{x+2}}$	Apply method of quadrature
$y(x) = \int u^{-1/2}\,du$	Let $u = x+2$
$y(x) = 2u^{1/2} + C$	power rule
$y(x) = 2\sqrt{x+2} + C$	
$-1 = 2\sqrt{2+2} + C$	use $y(2) = -1$
$C = -5$	
$y(x) = 2\sqrt{x+2} - 5$	

Check: agrees with textbook