

Math 2250  
Report Contents 2.4,2.5,2.6 Problem 6  
August 2006

## Exact Solution 2.4,2.5,2.6-#6

The exact solution for  $y' = -2xy$ ,  $y(0) = 2$  should be derived as a regular daily problem, submitted for grading in class. In the numerical work, this symbolic derivation is only referenced (do not derive again!). The answer:

$$y = 2e^{-x^2}.$$

A table of exact values is required in order to make comparison tables. Make this table for each problem separately, as the values used vary from one comparison to another.

## 2.4 Notes

### Numerical Solution 2.4-#6

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the Euler algorithm as applied to problem #6, then reference the worksheet for results.

The maple code referenced in the 19-page internet document *Numerical DE Manuscript* will be used. There is a text file of the actual code segments in the internet document document *Numerical DE maple coding hints*. Both are located at the course web site.

Sample Euler code:

```
# Warning: These snips of code made for y'=1-x-y, y(0)=3.
#           Code computes approx values for y(0.1) to y(0.5).
# 'Dots' is the list of dots for connect-the-dots graphics.
# =====
# Euler. Group 1, initialize.
f:=(x,y)->1-x-y;
x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:
# Group 2, repeat 5 times. Euler's method
for j from 1 to 5 do
  Y:=y0+h*f(x0,y0);
  x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
end do;
# Group 3, show Dots, then plot.
Dots[1],Dots[2],Dots[3],Dots[4],Dots[5],Dots[6];
plot([Dots]);
```

To start, get the sample code to produce correct answers to the example supplied in the text file source. Once correct, modify the code for #6. The step size  $h = 0.25$  produces a dot table of 3 rows, whereas the step size  $h = 0.1$  makes a dot table with 6 rows.

### Comparison Table 2.4-#6

The comparison will be 3 rows in 2.4-#6, which means half the  $h = 0.1$  data is not used in the report. The table should list  $x$ ,  $y_1$ ,  $y_2$ ,  $y$  where  $y_1$  is the  $h = 0.25$  approximate value,  $y_2$  is the  $h = 0.1$  approximate value and  $y$  is the exact value.

### Graphics 2.4-#6

There should be three graphics, one for  $h = 0.25$ , one for  $h = 0.1$  and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

## Appendix: Hand Solution Steps 2.4-#6

Include a derivation of the numerical values for line two of the dots table for each case  $h = 0.1$  and  $h = 0.05$ . Show all steps by hand. This is the only cross-check on the numerics.

## Appendix: Maple Worksheet 2.4-#6

Attach a print of the maple worksheet that contains all computer code and data used in 2.4-#6. Reference this appendix during the report.

## 2.5 Notes

### Numerical Solution 2.5-#6

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the Heun (modified Euler) algorithm as applied to problem #6, then reference the worksheet for results.

Sample Heun code:

```
# Warning: These snips of code made for y'=1-x-y, y(0)=3.
#           Code computes approx values for y(0.1) to y(0.5).
# 'Dots' is the list of dots for connect-the-dots graphics.
# =====
# Heun [=Modified Euler]. Group 1, initialize.
f:=(x,y)->1-x-y:
x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:
# Group 2, repeat 5 times. Heun's method
for j from 1 to 5 do
  Y1:=y0+h*f(x0,y0);
  Y:=y0+h*(f(x0,y0)+f(x0+h,Y1))/2:
  x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
end do:
# Group 3, show Dots, then plot.
Dots[1],Dots[2],Dots[3],Dots[4],Dots[5],Dots[6];
plot([Dots]);
```

To start, get the sample Heun code to produce correct answers to the example supplied in the text file source. Once correct, modify the code to apply to 2.5-#6. The step size  $h = 0.1$  produces a dot table of 6 rows.

### Comparison Table 2.5-#6

The comparison will be 6 rows in 2.5-#6. The table should list  $x$ ,  $y_1$ ,  $y$  where  $y_1$  is the  $h = 0.1$  approximate value and  $y$  is the exact value.

### Graphics 2.5-#6

There should be two graphics, one for  $h = 0.1$  and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

## Appendix: Hand Solution Steps 2.5-#6

Include a derivation of the numerical values for line two of the dots table for  $h = 0.1$ . Show all steps by hand. This is the only cross-check on the numerics.

## Appendix: Maple Worksheet 2.5-#6

Attach a print of the maple worksheet that contains all computer code and data used in 2.5-#6. Reference this appendix during the production of the report.

## 2.6 Notes

### Numerical Solution 2.6-#6

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the RK4 algorithm as applied to problem #6, then reference the worksheet for results.

Sample RK4 code:

```
# Warning: These snips of code made for  $y'=1-x-y$ ,  $y(0)=3$ .
#           Code computes approx values for  $y(0.25)$  to  $y(0.5)$ .
# 'Dots' is the list of dots for connect-the-dots graphics.
# =====
# RK4. Group 1, initialize.
f:=(x,y)->1-x-y:
x0:=0:y0:=3:h:=0.25:Dots:=[x0,y0]:
# Group 2, repeat one time. RK4 method
for j from 1 to 1 do
k1:=h*f(x0,y0):
k2:=h*f(x0+h/2,y0+k1/2):
k3:=h*f(x0+h/2,y0+k2/2):
k4:=h*f(x0+h,y0+k3):
Y:=y0+(k1+2*k2+2*k3+k4)/6:
x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0]:
end do:
# Group 3, show Dots, then plot.
Dots[1],Dots[2],Dots[3]:
plot([Dots]);
```

To start, get the sample RK4 maple code, referenced in the 19-page internet document *Numerical DE Manuscript*, to produce correct answers to the example supplied in the text file source. Once correct, modify the code for #6. The step size  $h = 0.25$  produces a dot table of 3 rows.

### Comparison Table 2.6-#6

The comparison will be 3 rows in 2.6-#6. The table should list  $x$ ,  $y_1$ ,  $y$  where  $y_1$  is the  $h = 0.25$  approximate value and  $y$  is the exact value.

### Graphics 2.6-#6

There should be two graphics, one for  $h = 0.25$  and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

### Appendix: Hand Solution Steps 2.6-#6

Skip this step for 2.6-#6, because the machine is likely more reliable than a hand calculation. Instead of a hand check, check the algorithm on several problems which have known solutions (do not submit a record of this check).

### Appendix: Maple Worksheet 2.6-#6

Attach a print of the maple worksheet that contains all computer code and data used in 2.6-#6. Reference this appendix during the production of the report.