

Math 2250
Report Contents 2.4,2.5,2.6 Problem 12
September 2004

Exact Solution 2.4,2.5,2.6-#12

The exact solution should be derived by hand just once for 2.4-#12, and then only referenced in 2.5-#12, 2.6-#12 (do not derive again!). The answer:

$$y = \frac{x - 4}{x - 2}.$$

A table of exact values is required in order to make comparison tables. Make this table for each problem separately, as the values used vary from one comparison to another.

2.4 Notes

Numerical Solution 2.4-#12

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the Euler algorithm as applied to problem #12, then reference the worksheet for results.

The maple code referenced in the 19-page internet document *Numerical DE Manuscript* will be used. There is a text file of the actual code segments in the internet document document *Numerical DE maple coding hints*. Both at the course web site.

To start, get the sample code to produce correct answers to the example supplied in the text file source. Once correct, modify the code for #12. The step size $h = 0.01$ produces a dot table of 101 rows, whereas the step size $h = 0.005$ makes a dot table with 201 rows. There is no reason to include all of this data in an appendix: see notes below.

The term *percentage error* is defined by

$$\text{Error} = 100 \frac{|approx - exact|}{|exact|}.$$

This equality is also called the percentage relative error.

Comparison Table 2.4-#12

Most of the $h = 0.01$ and $h = 0.005$ data is not used in the report. You must select the data to be printed in the table from the raw data produced in Maple. Loop control should be used to create the dots table, but the dots table should not be printed in its entirety in the final report.

Please do not print data that is not used, as it makes the reports too long and it wastes paper. Observe that the percentage error can be computed from the 6-item table: find just the error values for $x = 0, 0.2, 0.4, 0.6, 0.8, 1.0$.

The comparison will be 6 rows in 2.5-#12. The table should list $x, y_1, y_2, y, E1, E2$ where y_1 is the $h = 0.01$ approximate value, y_2 is the $h = 0.005$ approximate value, y is the exact value, $E1$ is the percentage error for $h = 0.01$ and $E2$ is the percentage error for $h = 0.005$. The x -values in the table are 0.0, 0.2, 0.4, 0.6, 0.8, 1.0.

Graphics 2.4-#12

There should be three graphics, one for $h = 0.01$, one for $h = 0.005$ and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

Appendix: Hand Solution Steps 2.4-#12

Include a derivation of the numerical values for line two of the dots table for each case $h = 0.01$ and $h = 0.005$. Show all steps by hand. This is the only cross-check on the numerics.

Appendix: Maple Worksheet 2.4-#12

Attach a print of the maple worksheet that contains all computer code and data used in 2.4-#12. Reference this appendix during the report.

2.5 Notes

Numerical Solution 2.5-#12

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the Heun (modified Euler) algorithm as applied to problem #12, then reference the worksheet for results.

To start, get the sample Heun code to produce correct answers to the example supplied in the text file source. Once correct, modify the code to apply to 2.5-#12. The step size $h = 0.01$ produces a dot table of 101 rows.

Comparison Table 2.5-#12

The comparison will be 6 rows in 2.5-#12. The table should list x , y_1 , y_2 , y , E_1 , E_2 where y_1 is the $h = 0.01$ approximate value, y_2 is the $h = 0.005$ approximate value, y is the exact value, E_1 is the percentage error for $h = 0.01$ and E_2 is the percentage error for $h = 0.005$. The x -values in the table are 0.0, 0.2, 0.4, 0.6, 0.8, 1.0.

Graphics 2.5-#12

There should be three graphics, one for $h = 0.01$, one for $h = 0.005$ and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

Appendix: Hand Solution Steps 2.5-#12

Include a derivation of the numerical values for line two of the dots table for $h = 0.01$. Show all steps by hand. Don't show any $h = 0.005$ steps. This is the only cross-check on the numerics.

Appendix: Maple Worksheet 2.5-#12

Attach a print of the maple worksheet that contains all computer code and data used in 2.5-#12. Reference this appendix during the production of the report.

2.6 Notes

Numerical Solution 2.6-#12

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the RK4 algorithm as applied to problem #12, then reference the worksheet for results.

To start, get the sample RK4 maple code, referenced in the 19-page internet document *Numerical DE Manuscript*, to produce correct answers to the example supplied in the text file source. Once correct, modify the code for #12. The step size $h = 0.2$ produces a dot table of 6 rows. For $h = 0.1$ there are 11 rows.

Comparison Table 2.6-#12

The comparison will be 6 rows in 2.6-#12. The table should list x , y_1 , y_2 , y , E_1 , E_2 where y_1 is the $h = 0.2$ approximate value, y_2 is the $h = 0.1$ approximate value, y is the exact value, E_1 is the percentage error for $h = 0.2$ and E_2 is the percentage error for $h = 0.1$. The x -values in the table are 0.0, 0.2, 0.4, 0.6, 0.8, 1.0.

Graphics 2.6-#12

There should be three graphics, one for $h = 0.2$, one for $h = 0.1$ and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

Appendix: Hand Solution Steps 2.6-#12

Skip this step for 2.6-#12, because the machine is likely more reliable than a hand calculation. Instead of a hand check, check the algorithm on several problems which have known solutions.

Appendix: Maple Worksheet 2.6-#12

Attach a print of the maple worksheet that contains all computer code and data used in 2.6-#12. Reference this appendix during the production of the report.