Name. ______________________ **Class time**. _________

Applied Differential Equations 2250-1 and 2250-2

Midterm Exam 4 Version A-D

Submit problem 1 on Nov 26

Submit problem 2 on Dec 1

Submit problems 3,4 on Dec 10 to 113JWB

Instructions. The take-home problems below are to be submitted at class time on the due dates listed above. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

The in-class portion of the exam (December 3) is 15 minutes, one problem, of a type similar to one of the problems. Books, calculators and notes are not allowed.

1. **(Eigenanalysis)** A cross-coupled system $\mathbf{x}' = A\mathbf{x}$ and a diagonal system $\mathbf{y}' = D\mathbf{y}$ are related by a change of variables $\mathbf{x} = P\mathbf{Y}$ where P is invertible and 3×3. Known are

$$A = \begin{pmatrix} -2 & -12 & -5 \\ 0 & 5 & 0 \\ 0 & -2 & 3 \end{pmatrix}, \quad D = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$$

Represent the general solution of $\mathbf{x}' = A\mathbf{x}$ as a matrix product, by solving $\mathbf{y}' = D\mathbf{y}$ and then back-substituting the answer into the relation $\mathbf{x} = P\mathbf{y}$. Part of the solution is to find P explicitly. An answer check is expected.

Please staple this exam to your solutions and submit it at class time on the due date.

Name. ________________________ **Class time.** __________

Applied Differential Equations 2250-1 and 2250-2

Midterm Exam 4 Version A-D

Submit problem 2 on Dec 1

Submit problems 3,4 on Dec 10 to 113JWB

Instructions. The take-home problems below are to be submitted at class time on the due dates listed above. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

The in-class portion of the exam (December 3) is 15 minutes, one problem, of a type similar to one of the problems. Books, calculators and notes are not allowed.

2. (Coupled spring-mass system) Consider the railway car system (Edwards-Penney page 429) $M\mathbf{x}'' = K\mathbf{x}$ where

$$K = \begin{pmatrix} -k_1 - k_2 & k_2 & 0 \\ k_2 & -k_2 - k_3 & k_3 \\ 0 & k_3 & -k_3 - k_4 \end{pmatrix}, \quad M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{1}$$

Let $k_1 = 0$, $k_2 = 4000$, $k_3 = 4000$, $k_4 = 0$, $m_1 = 1000$, $m_2 = 500$, $m_3 = 1000$. Find the natural frequencies of oscillation ω_1, ω_2, ω_3 for system (1). *Solve* for $\mathbf{x}(t)$, showing all eigenanalysis steps.

Please staple this exam to your solutions and submit it at class time on the due date.

Name. ______________________ **Class time**. _________

Applied Differential Equations 2250-1 and 2250-2

Midterm Exam 4 Version A-D

Submit problems 3,4 on Dec 10 to 113JWB

Instructions. The take-home problems below are to be submitted on December 10 to my office 113 jwb, before noon. If no one is there, then kindly slide the exam under the door.

3. **(Three methods)** The solution $x(t)$ of the problem

$$x''(t) + 3x'(t) + 2x(t) = te^{-2t} + \sin t, \quad x(0) = 0, \quad x'(0) = 0$$

is given by

$$x(t) = \frac{3}{2}\,e^{-t} - \frac{1}{2}t^2e^{-2t} - te^{-2t} - \frac{6}{5}\,e^{-2\,t} - \frac{3}{10}\,\cos(t) + \frac{1}{10}\,\sin(t).$$

Verify the solution by displaying the steps for three methods: (1) Undetermined coefficients, (2) variation of parameters and (3) Laplace transform. Attach an appendix for `Maple` or handwritten integration detail in (2).

4. **(Laplace inverse transform)** Display the partial fraction steps involved in solving for $f(t)$ in the Laplace equation

$$\mathcal{L}(f(t)) = \frac{4\,s^4 + 21\,s^2 - 12\,s^3 - 88\,s + 100}{(s+1)\,(s-2)^2\,(s^2+4)}.$$

Please flag the step where Lerch's theorem is applied to give the answer

$$f(t) = 5\,e^{-t} - te^{2\,t} - \cos(2\,t) + 3\,\sin(2\,t).$$

Please staple this exam to your solutions and submit it by noon on the due date.