Name. _____ **Section**. _____

Applied Differential Equations 2250-1 and 2250-2
Midterm Exam 3, Due classtime 7 April 2003

Instructions. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

The in-class portion of the exam on April 4 is 15 minutes, one problem, of a type similar to one of the four problems. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. **(Variation of Parameters)** Show the steps in the solution of

$$y'' - y = -xe^{-x}$$

by variation of parameters, to obtain the general solution

$$y = c_1 e^x + c_2 e^{-x} + (1/4)x(1+x)e^{-x}.$$

Use formula (3), page 335.

2. **(Undetermined Coefficients)** Show the steps in the solution of

$$y''' - y' = x^2 + xe^{-x} + \sin x$$

by undetermined coefficients, to obtain the general solution

$$y = c_1 e^x + c_2 e^{-x} + c_3 - (1/3)x^3 - 2x + (1/2)\cos(x) + (1/4)e^{-x}x^2 + (3/4)xe^{-x} + (7/8)e^{-x}.$$

3. **(Practical Resonance)** Given $x'' + 10x' + 50x = 16\cos(\omega t)$, find
   (a) the derivative of the amplitude $C(\omega)$,
   (b) the practical resonant frequency ω^*.
   Use formulas on pages 346–347. Show all steps used to obtain the answers.

4. **(Spring-mass system)** Find a spring-mass equation $mx'' + cx' + kx = F(t)$ whose general solution is given by

$$x(t) = c_1 e^{-t}\cos(\sqrt{2}t) + c_2 e^{-t}\sin(\sqrt{2}t) + \sin(2t) + 5\cos(2t).$$

Hint: Apply superposition to determine x_h and x_p. Let $m = 1$. Choose c and k to match x_h. Compute $F(t)$ from the differential equation with x replaced by x_p.