Applied Differential Equations 2250
Sample Midterm Exam 2, 7:30 and 10:45
Monday, 28 March 2005

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (rref)

(a) Determine b such that the system has infinitely many solutions:

$$\begin{array}{rcrcrcl} x &+& 2y &+& z &=& b \\ 3x &+& y &+& 2z &=& 2b \\ 4x &+& 3y &+& 3z &=& 1+b \end{array}$$

(b) Determine a, b such that the system has infinitely many solutions:

$$\begin{array}{rcrcrcl} x &+& 2y &+& z &=& a \\ 5x &+& y &+& 2z &=& 3a \\ 6x &+& 3y &+& bz &=& 1+a \end{array}$$

2. (vector spaces)

(a) Give two examples of vector spaces of functions, one of dimension two and one of dimension three.

(b) Let V be the vector space of all continuous functions on the real line and let S be the subset of V given by all solutions of the differential equation $y' = -2y$. Prove that S is a subspace of V.

(c) Find a basis for the subspace of $\mathcal{R}^3$ given by the system of equations

$$\begin{array}{rcl} x + 2y - z &=& 0, \\ x + y - 2z &=& 0, \\ y + z &=& 0, \end{array}$$

3. (independence)

(a) Let $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{w} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. State and apply a test that shows $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are dependent.

(b) Extract from the list below a largest set of independent vectors.

$$\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ -2 \\ 0 \\ -2 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 3 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{d} = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 4 \end{pmatrix}, \mathbf{e} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 3 \end{pmatrix}.$$

4. (determinants and elementary matrices)

(a)Assume given 3×3 matrices A, B. Suppose $B = E_1 E_2 A$ and E_1, E_2 are elementary matrices representing swap rules. Explain precisely why $\det(B) = \det(A)$.

(b) Let A and B be two 7×7 matrices such that AB contains two duplicate rows. Explain precisely why either $\det(A)$ or $\det(B)$ is zero.

5. (inverses and Cramer's rule)

(a) Determine all values of x for which A^{-1} exists: $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & -3 \\ 0 & x & 1 \end{pmatrix}$.

(b) Solve for y in $A\mathbf{u} = \mathbf{b}$ by Cramer's rule: $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 0 & 2 \\ 2 & -2 & 1 \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.