Name ______________________ **Class time** _________

Applied Differential Equations 2250-1 and 2250-2
Midterm Exam 2, Fall 2003, Version T-Z

Due Wed 15 Oct (1,2) and Fri 17 Oct (3,4)

Inclass Exam Date: Monday, 20 October, 2003

Instructions. Choose the exam version based upon your last name, e.g., John Timpson chooses exam version T-Z, because **T** of **T**impson is between **T** and **Z**.

The four problems below are take-home, due on the dates above at class time. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output. The remaining 20% of the exam is in class, the last 15 minutes of the hour, one problem, of a type similar to # 3 or 4 below. No books, notes, calculators, computers or outside materials allowed.

1. **(Periodic harvesting)** The population equation $y' = 3y(4-y) - 11\sin(2\pi t)$ appears to have a steady-state periodic solution that oscillates about $y = 4$. (a) Apply ideas from the example below to make a computer graphic with 6 solution curves that oscillate about $y = 4$. Submit the plot and the maple code. (b) Find by computer experiment a threshold population size y_1 so that $y(0) < y_1$ implies $y(t) = 0$ (population dies out) for some later time t, while $y(0) > y_1$ implies $y(t) > 0$ forever and the solution $y(t)$ oscillates about $y = 3$. See Figure 2.5.12, page 128.

```
# Example.  See Figure 12, section 2.5
with(DEtools):
de:=diff(y(t),t)=y(t)*(2-y(t))-4*cos(4*Pi*t):
ic:=[y(0)=1.7],[y(0)=2],[y(0)=2.4],[y(0)=2.8]:
DEplot(de,y(t),t=0..4,y=1..3,[ic],stepsize=0.05);
```

2. **(Jules Verne Problem)** Assume a model

$$\frac{d^2r}{dt^2} = -\frac{Gm_1}{(R_1+r)^2} + \frac{Gm_2}{(R_2-R_1-r)^2}, \quad r(0) = 0, \quad r'(0) = v_0,$$

where R_2 is the mean center-to-center distance from the earth to the moon and R_1 is the mean radius of the earth. The mass m_1 of the earth and m_2 of the moon appear, plus the universal gravitation constant G. All units are MKS.

(a) Explain why this model takes into account the gravitational attraction of both the moon and the earth.

(b) Calculate the distance r^* at which the projectile has net acceleration zero. Give a symbolic answer and also a numerical answer $\approx 3.39 \times 10^8$ meters.

(c) Conduct a numerical experiment to find the flight time to the moon, when the launch velocity $r'(0)$ is 32 m/s faster than the minimal launch velocity $v_0 = \sqrt{2F(0) - 2F(r^*)}$, $F(r) = \dfrac{Gm_1}{R_1 + r} + \dfrac{Gm_2}{R_2 - R1 - r}$. Use the sample maple code below to do the experiment.

```
# Group 1
G:=6.6726e-11:  m1:=5.975e24:  m2:=7.36e22:
R1:=6.378e6:  R2:=3.84e8:  v0:=1000:  T:=210:
de:=diff(r(t),t,t)=-G*m1/(r(t)+R1)^2+G*m2/(R2-R1-r(t))^2:
ic:=r(0)=0,D(r)(0)=v0:
p:=dsolve({de,ic},r(t),type=numeric,method=lsode);
Y:=t->rhs(p(t)[2]):
plot('Y(t)',t=0..T);
# Plot done.  Change v0, T and re-execute group 1.
```

3. (Gaussian algorithm) Solve for x, y, z in the 3×3 linear system

$$\begin{array}{rcrcrcr} 2x & + & 2(a-b)y & + & cz & = & -b \\ 3x & + & (a-b)y & + & 2cz & = & 2b \\ 5x & + & 3(a-b)y & + & 3cz & = & b \end{array}$$

using the Gaussian algorithm, for all constant values of a, b, c. Include all algorithm details and an **answer check** for each of the three separate cases. Sanity check: $a + b \neq 0$ is one case, with parametric solution $x = 5b/4 - 3ct_1/4$, $y = -7b/(4a - 4b) + ct_1/(4a - 4b)$, $z = t_1$. The case $a - b = 0$ has subcases $c \neq 0$ and $c = 0$, for one of which you will report *no solution*.

4. (Inverse matrix) Determine by `rref` methods the inverse matrix of

$$A = \begin{pmatrix} 1 & b & 0 \\ -a & 0 & -b \\ 0 & 1 & 1 \end{pmatrix}.$$

Please state conditions on a, b for when the inverse exists. Show all hand details. Prove that in the absence of your condition, no inverse exists. Include an **answer check**, preferably done in `maple`.