

Name \_\_\_\_\_

Class time \_\_\_\_\_

## Applied Differential Equations 2250-1 and 2250-2

### Midterm Exam 2, Fall 2003, Version A-D

Due Wed 15 Oct (1,2) and Fri 17 Oct (3,4)

Inclass Exam Date: Monday, 20 October, 2003

**Instructions.** Choose the exam version based upon your last name, e.g., John Charles chooses exam version A-D, because **C** of Charles is between **A** and **D**.

The four problems below are take-home, due on the dates above at class time. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output. The remaining 20% of the exam is in class, the last 15 minutes of the hour, one problem, of a type similar to # 3 or 4 below. No books, notes, calculators, computers or outside materials allowed.

- (Periodic harvesting)** The population equation  $y' = 4y(3 - y) - 7 \sin(2\pi t)$  appears to have a steady-state periodic solution that oscillates about  $y = 3$ . (a) Apply ideas from the example below to make a computer graphic with 6 solution curves that oscillate about  $y = 3$ . Submit the plot and the `maple` code. (b) Find by computer experiment a threshold population size  $y_1$  so that  $y(0) < y_1$  implies  $y(t) = 0$  (population dies out) for some later time  $t$ , while  $y(0) > y_1$  implies  $y(t) > 0$  forever and the solution  $y(t)$  oscillates about  $y = 3$ . See Figure 2.5.12, page 128.

```
# Example. See Figure 12, section 2.5
with(DEtools):
de:=diff(y(t),t)=y(t)*(2-y(t))-4*cos(4*Pi*t):
ic:=[y(0)=1.7],[y(0)=2],[y(0)=2.4],[y(0)=2.8]:
DEplot(de,y(t),t=0..4,y=1..3,[ic],stepsize=0.05);
```

- (Jules Verne Problem)** Assume a model

$$\frac{d^2r}{dt^2} = -\frac{Gm_1}{(R_1 + r)^2} + \frac{Gm_2}{(R_2 - R_1 - r)^2}, \quad r(0) = 0, \quad r'(0) = v_0,$$

where  $R_2$  is the mean center-to-center distance from the earth to the moon and  $R_1$  is the mean radius of the earth. The mass  $m_1$  of the earth and  $m_2$  of the moon appear, plus the universal gravitation constant  $G$ . All units are *MKS*.

(a) Explain why this model takes into account the gravitational attraction of both the moon and the earth.

(b) Calculate the distance  $r^*$  at which the projectile has net acceleration zero. Give a symbolic answer and also a numerical answer  $\approx 3.39 \times 10^8$  meters.

(c) Conduct a numerical experiment to find the flight time to the moon, when the launch velocity  $r'(0)$  is 20 m/s faster than the minimal launch velocity  $v_0 = \sqrt{2F(0) - 2F(r^*)}$ ,  $F(r) = \frac{Gm_1}{R_1 + r} + \frac{Gm_2}{R_2 - R_1 - r}$ . Use the sample maple code below to do the experiment.

```
# Group 1
G:=6.6726e-11: m1:=5.975e24: m2:=7.36e22:
R1:=6.378e6: R2:=3.84e8: v0:=1000: T:=210:
de:=diff(r(t),t,t)=-G*m1/(r(t)+R1)^2+G*m2/(R2-R1-r(t))^2:
ic:=r(0)=0,D(r)(0)=v0:
p:=dsolve({de,ic},r(t),type=numeric,method=lsode);
Y:=t->rhs(p(t)[2]):
plot('Y(t)',t=0..T);
# Plot done. Change v0, T and re-execute group 1.
```

3. **(Gaussian algorithm)** Solve for  $x, y, z$  in the  $3 \times 3$  linear system

$$\begin{aligned} 2x + 2(a+b)y + cz &= b \\ 3x + (a+b)y + 2cz &= 2b \\ 5x + 3(a+b)y + 3cz &= 3b \end{aligned}$$

using the Gaussian algorithm, for all constant values of  $a, b, c$ . Include all algorithm details and an **answer check** for each of the three separate cases. Sanity check:  $a+b \neq 0$  is one case, with parametric solution  $x = 3b/4 - 3ct_1/4$ ,  $y = -b/(4a+4b) + ct_1/(4a+4b)$ ,  $z = t_1$ . The case  $a+b = 0$  has subcases  $c \neq 0$  and  $c = 0$ , for one of which you will report *no solution*.

4. **(Inverse matrix)** Determine by rref methods the inverse matrix of

$$A = \begin{pmatrix} 1 & b & 0 \\ a & 0 & b \\ 0 & 1 & 1 \end{pmatrix}.$$

Please state conditions on  $a, b$  for when the inverse exists. Show all hand details. Prove that in the absence of your condition, no inverse exists. Include an **answer check**, preferably done in maple.