Name. _____ **Section**. _____

Applied Differential Equations 2250-1 and 2250-2
Midterm Exam 1, Spring 2003
Exam Date: Wednesday, 29 January, 2003

Instructions. The four problems below are take-home, due on the date above at class time. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

The remaining 20% of the exam is in class, 15 minutes, one problem, of a type similar to one of the problems below. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. **(Quadrature Equations)** Solve by the method of quadrature the initial value problem $y''' = x^2 - xe^{-2x} - \sin 3x$, $y(0) = y'(0) = 0$, $y''(0) = 1$. Show all integration steps, by hand. An answer check is required.

2. **(Separable Equations)** Solve the separable problem for equilibrium and non-equilibrium solutions. Identify the *implicit* solution. Find, if possible, an *explicit* solution. Check all answers.
$$2y' = \cos x - \cot x - 16y^2(1 - \csc x)\cos x.$$

3. **(Linear Equations)** Solve the linear equation $2xy'(x) + 5y(x) = \sqrt{x}e^{-2x}$, $y(1) = 4$. Expected details include the factorization method and all integration steps, by hand. An answer check is required.

4. **(Application: Torricelli's law)** A water tank has the shape of $y = x^{6/5}$ revolved around the y-axis. The depth of the water is 20 feet. After 1 hour and 15 minutes, the drain hole at $(0, 0)$ empties the tank to a depth of 12 feet. Find the additional time it takes to empty the tank. Ref: Exercise 1.4-52 and equation 1.4-(24).