

Math 2250 Maple Lab 4, February 2006. Mechanical Oscillations.

NAME \_\_\_\_\_ CLASSTIME \_\_\_\_ VERSION A-E, F-K, L-R, S-Z

Circle the version - see problem L4.1. There are three (3) problems in this project. Please answer the questions A, B, C , ... associated with each problem. The original worksheet "2250mapleL4-S2006.mws" is a template for the solution; you must fill in the code and all comments. Sample code can be copied with the mouse. Use pencil freely to annotate the worksheet and to clarify the code and figures.

The problem headers for the Spring 2006 version of Mechanical Oscillations maple lab 4.

\_\_\_\_\_L4.1. UNDER-DAMPED FREE OSCILLATIONS.  
\_\_\_\_\_L4.2. UNDAMPED FORCED OSCILLATIONS.  
\_\_\_\_\_L4.3. PRACTICAL RESONANCE.

#### L4.1. PROBLEM (UNDER-DAMPED FREE OSCILLATIONS)

FREE OSCILLATIONS. Consider the problem of free linear oscillations

$$m x'' + c x' + k x = 0, \\ x(0) = 0, \quad x'(0) = 1.$$

Here,  $m$ ,  $c$  and  $k$  are non-negative constants. The under-damped case is studied here,  $c^2 < 4km$ , as on page 327 in E&P. Depending on the first letter of your last name, assume:

Version A-E: $m=1$ , $c=4$	Version F-K: $m=2$ , $c=4$
Version L-R: $m=3$ , $c=5$	Version S-Z: $m=4$ , $c=5$

- Display a Hooke's constant  $k > 0$  so that the solution  $x(t)$  is under-damped. Check that  $x(t) = 0$  for infinitely many  $t > 0$ . Display the exact solution  $x(t)$  obtained by maple methods as in the example below.
- Plot the exact symbolic solution  $x(t)$  on a suitable  $t$ -interval. Check the graphic against Figure 5.4.9 page 328 of E&P.
- Estimate from the graph the decimal value of the pseudo-period. Display the graphical estimate and also the exact pseudo-period  $2\pi/\omega$ , where  $\omega$  is the natural frequency of the trigonometric term in the solution  $x(t)$  found in item 2.4.A.

EXAMPLE(Wrong parameters! Change it!)	# Use semicolons to see what you have done.
de:=3*diff(x(t),t,t)+1.5*diff(x(t),t)+4*x(t)=0:	# Define the differential equation
solve(3*r^2+1.5*r+4=0,r);	# Solve the characteristic equation.
ic:=x(0)=0,D(x)(0)= 1:	# Define the initial conditions
p:=dsolve({de,ic},x(t),method=laplace):	# Symbolically solve for x(t)
X:=unapply(rhs(p),t):	# Capture the dsolve symbolic
	# answer as a function X(t)
plot(X(t),t=0..5);	# Plot the solution

Maple tip: Click with the mouse on the graphic to print the cursor

location (left upper corner of the maple window). The coordinates printed are of the form (x,y). From this coordinate information, a simple subtraction estimates the period.

```
>
> #L4.1-A Define k, then solve.
> # under-damped means  $mr^2+cr+k=0$  has two conjugate complex roots.
> #L4.1-B Plot.
> #L4.1-C Pseudo-period calculations.
>
```

#### L4.2. PROBLEM (UNDAMPED FORCED OSCILLATIONS )

FORCED LINEAR OSCILLATIONS. Consider the undamped ( $c=0$ ) forced problem

$$mx'' + kx = 5 \cos(\omega t), \\ x(0)=0, \quad x'(0)=0,$$

where  $m$ ,  $k$  and  $\omega$  are non-negative constants. Depending on the first letter of your last name, assume:

Version A-E: $m=1$ , $k=3.5$	Version F-K: $m=2$ , $k=2.5$
Version L-R: $m=3$ , $k=4.5$	Version S-Z: $m=4$ , $k=4.5$

- Choose the forcing angular frequency  $\omega$  to be 3 times larger than the natural angular frequency  $\omega_0$ ,  $\omega_0^2=k/m$ . Solve for  $x(t)$  using `dsolve()`. Plot the solution  $x(t)$  on a suitable interval in order to show the global behavior of the solution  $x(t)$ . See Figure 5.6.2, page 350.
- The solution  $x(t)$  is the sum of two functions, one of period  $2\pi/\omega$  and the other of period  $2\pi/\omega_0$ . Display the exact period, as calculated from the solution formula for  $x(t)$  -- see page 350 for details.
- Suggest a value for the forcing frequency  $\omega$  so that the oscillations exhibit resonance. Show resonant behavior on a graph. Check against Figure 5.6.4, page 352.

```
> #L4.2-A
> #L4.2-B
> #L4.2-C
>
```

#### L4.3. PROBLEM (PRACTICAL RESONANCE)

Consider the damped forced problem

$$mx'' + c x' + kx = 5 \cos(\omega t), \\ x(0)=0, \quad x'(0)=0.$$

Depending on the first letter of your last name, assume:

Version A-E: $m=1$ , $k=30$	Version F-K: $m=2$ , $k=36$
Version L-R: $m=3$ , $k=45$	Version S-Z: $m=4$ , $k=55$

- Consider the damping constants  $c=2$ ,  $c=1$  and  $c=1/2$ . Compute the amplitude function  $C(\omega)$  [page 357] for these three equations,

then plot for  $w=0$  to  $w=20$  the three amplitude graphs on a single set of axes. Compare against Figure 5.6.9 page 357 of E&P (it has one curve, yours has 3 curves).

- B. For each case  $c=2$ ,  $c=1$ ,  $c=1/2$ , print the values  $w^*$ ,  $C^*$  where  $C^*=C(w^*)=\max \{C(w) : 0 \leq w \leq 20\}$ . The three data pairs should show that  $C^*$  becomes larger as  $c$  tends to zero. SAVE YOUR MAPLE FILE FREQUENTLY

Maple Hint: Use Maple's mouse interface on the graphic of Part C. Specifically, click on a possible maximum (horizontal tangent) in the graph to display the values  $w^*$ ,  $C^*$  on the screen. Copy the values on paper.

```
EXAMPLE(Beware! Wrong values!)
F:=15: m:=1: k:=25: c:='c': w:='w':
C:=(w,c)->F/sqrt((k-m*w*w)^2+(c*w)^2):
plot({C(w,4),C(w,3),C(w,2)},w=0..15,color=black);
```

```
> #L4.3-A Plot C(w), three graphics on one set of axes
> #L4.3-B Table of six data values for w*, C*
```