Due date: See the internet due dates. Maple lab 3 has problems L3.1, L3.2, L3.3.


Problem L3.1. (Matrix Algebra)

Define \( A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \), \( B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} \), \( v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \) and \( w = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \). Create a worksheet in maple which states this problem in text, then defines the four objects. The worksheet should contain text, maple code and displays. Continue with this worksheet to answer (1)–(7) below. Submit problem L3.1 as a worksheet printed on 8.5 by 11 inch paper. See Example 1 for maple commands.

(1) Compute \( AB \) and \( BA \). Are they the same?

(2) Compute \( A + B \) and \( B + A \). Are they the same?

(3) Let \( C = A + B \). Compare \( C^2 \) to \( A^2 + 2AB + B^2 \). Explain why they are different.

(4) Compute the transpose of \( AB \) and compare it to the product of the transpose of \( A \) with the transpose of \( B \), multiplied in the correct order so that you expect equality.

(5) Solve for \( X \) in \( BX = v \) by three different methods.

(6) Solve \( AY = v \) for \( Y \). Do an answer check.

(7) Solve \( AZ = w \). Explain your answer.

Problem L3.2. (Row space)

Let \( A = \begin{pmatrix} 1 & 1 & 1 & 2 & 6 \\ 2 & 3 & -2 & 1 & -3 \\ 0 & 1 & -4 & -3 & -15 \\ 1 & 2 & -3 & -1 & -9 \end{pmatrix} \). Find three different bases for the row space of \( A \), using the following methods.

1. The method of Example 2, below.

2. The maple command \texttt{rowspace(A)}

3. The \texttt{rref}-method: select rows from \texttt{rref(A)}.

Verify that all three bases are equivalent.

Problem L3.3. (Matrix Equations)

Let \( A = \begin{pmatrix} 8 & 10 & 3 \\ -3 & -5 & -3 \\ -4 & -4 & 1 \end{pmatrix} \), \( T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{pmatrix} \). Let \( P \) denote a \( 3 \times 3 \) matrix. Assume the following result (proved below):

\textbf{Lemma 1.} The equality \( AP = PT \) holds if and only if the columns \( v_1, v_2, v_3 \) of \( P \) satisfy \( Av_1 = v_1, \, Av_2 = -2v_2, \, Av_3 = 5v_3 \).

(a) Determine three specific columns for \( P \) such that \( \det(P) \neq 0 \) and \( AP = PT \). Infinitely many answers are possible. See Example 4 for the maple method that determines a column of \( P \).

(b) After reporting the three columns, check the answer by computing \( AP - PT \) (it should be zero) and \( \det(P) \) (it should be nonzero).

Staple this page on top of the maple work sheets. Examples on the next page …
Example 1. Let \( A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 0 & -1 \end{pmatrix} \) and \( b = \begin{pmatrix} 9 \\ 8 \\ 3 \end{pmatrix} \). Create a maple work sheet. Define and display matrix \( A \) and vector \( b \). Then compute

(1) The inverse of \( A \).

(2) The augmented matrix \( C = \text{aug}(A, b) \).

(3) The reduced row echelon form \( R = \text{rref}(C) \).

(4) The column \( X \) of \( R \) which solves \( AX = b \).

(5) The matrix \( A^3 \).

(6) The transpose of \( A \).

(7) The matrix \( A - 3A^2 \).

(8) The solution \( X \) of \( AX = b \) by two methods different than (4).

Solution: A lab instructor can help you to create a blank work sheet in maple, enter code and print the work sheet. The code to be entered appears below. To get help, enter \texttt{?linalg} into a worksheet, then select commands that match ones below.

\begin{verbatim}
with(linalg):
A:=matrix([[1,2,3],[2,-1,1],[3,0,-1]]);
b:=vector([9,8,3]);
print("(1)"); inverse(A);
print("(2)"); C:=augment(A,b);
print("(3)"); R:=rref(C);
print("(4)"); X:=col(R,4);
print("(5)"); evalm(A^3);
print("(6)"); transpose(A);
print("(7)"); evalm(A-3*(A^2));
print("(8)"); X:=linsolve(A,b); X:=evalm(inverse(A) &* b);
\end{verbatim}

Example 2. Let \( A = \begin{pmatrix} 1 & 1 & 1 & 2 & 6 \\ 2 & 3 & -2 & 1 & -3 \\ 3 & 5 & -5 & 1 & -8 \\ 4 & 3 & 8 & 2 & 3 \end{pmatrix} \).

(1) Find a basis for the column space of \( A \).

(2) Find a basis for the row space of \( A \).

(3) Find a basis for the nullspace of \( A \).

(4) Find \texttt{rank}(A) and \texttt{nullity}(A).

(5) Find the dimensions of the nullspace, row space and column space of \( A \).

Solution: The theory applied: The columns of \( B \) corresponding to the leading ones in \texttt{rref}(B) are independent and form a basis for the column space of \( B \). These columns are called the pivot columns of \( B \). Results for the row space can be obtained by applying the above theory to the transpose of the matrix.

The maple code which applies is

\begin{verbatim}
with(linalg):
A:=matrix([[1, 1, 1, 2, 6],
[2, 3, -2, 1, -3],
[3, 5, -5, 1, -8],
[4, 3, 8, 2, 3]]);
print("(1)"); C:=rref(A); # leading ones in columns 1,2,4
\end{verbatim}
The definition of matrix multiplication implies that \( AP \) always has a solution.

**Proof of Lemma 1**

Let \( A \) be a matrix. Define \( r_1 = 1, r_2 = -2, r_3 = 5 \). Assume \( AP = PT \), \( P = \text{aug}(v_1, v_2, v_3) \) and \( T = \text{diag}(r_1, r_2, r_3) \). Then \( AP = PT \) holds if and only if the columns of the two matrices match, which is equivalent to the three equations \( Av_1 = r_1v_1, Av_2 = r_2v_2, Av_3 = r_3v_3 \). The proof is complete.

**End of Maple Lab 3: Linear Algebra.**