The Basic Trial Solution Method. Outlined here is the method for a second order differential equation $ay'' + by' + cy = f(x)$. The method applies unchanged for $n$th order equations.

Step 1. Repeatedly differentiate the atoms of $f(x)$ until no new atoms appear. Collect the distinct atoms so found into a list of $k$ atoms. Multiply these atoms by undetermined coefficients $d_1, d_2, \ldots, d_k$, then add, defining trial solution $y$.

Step 2. Substitute $y$ into the differential equation.

Fixup Rule I. If some variable $d_p$ is missing in the equation, then step 2 fails. Correct the trial solution as follows. Variable $d_p$ appears in $y$ as term $d_pA$, where $A$ is an atom. Multiply $A$ and all its related atoms $B$ by $x$. The modified expression $y$ is called a corrected trial solution. Repeat step 2 until the equation contains all $k$ variables.

Step 3. Match coefficients of atoms left and right to write out linear algebraic equations for $d_1, d_2, \ldots, d_k$. Solve the equations for the unique solution.

Step 4. The corrected trial solution $y$ with evaluated coefficients $d_1, d_2, \ldots, d_k$ becomes the particular solution $y_p$. 

26
Symbols. The symbols $c_1$, $c_2$ are reserved for use as arbitrary constants in the general solution $y_h$ of the homogeneous equation. Symbols $d_1$, $d_2$, $d_3$, ... are reserved for use in the trial solution $y$ of the non-homogeneous equation. Abbreviations: $c = \text{constant}$, $d = \text{determined}$.

Superposition. The relation $y = y_h + y_p$ suggests solving $ay'' + by' + cy = f(x)$ in two stages:

(a) Apply the linear equation recipe to find $y_h$.

(b) Apply the basic trial solution method to find $y_p$.

We expect to find two arbitrary constants $c_1$, $c_2$ in the solution $y_h$, but in contrast, no arbitrary constants appear in $y_p$. Calling $d_1$, $d_2$, $d_3$, ... undetermined coefficients is misleading, because in fact they are eventually determined.
**Fixup rule II.** The rule predicts the corrected trial solution $y$ without having to substitute $y$ into the differential equation.

- Write down $y_h$, the general solution of homogeneous equation $ay'' + by' + cy = 0$, having arbitrary constants $c_1, c_2$. Create the corrected trial solution $y$ iteratively, as follows.

- Cycle through each term $d_p A$, where $A$ is an atom. If $A$ is also an atom appearing in $y_h$, then multiply $d_p A$ and each related atom term $d_q B$ by $x$. Other terms appearing in $y$ are unchanged.

- Repeat until each term $d_p A$ has atom $A$ distinct from all atoms appearing in homogeneous solution $y_h$. The modified expression $y$ is called the *corrected trial solution*.
Fixup rule III. The rule predicts the corrected trial solution $y$ without substituting it into the differential equation. This iterative algebraic method uses the roots of the characteristic equation to create $y$.

- Write down the roots of the characteristic equation. Let $L$ denote the list of distinct atoms for these roots.

- Cycle through each term $d_pA$, where $A$ is a atom. If $A$ appears in list $L$, then multiply $d_pA$ and each related atom term $d_qB$ by $x$. Other terms appearing in $y$ are unchanged.

- Repeat until the atom $A$ in an arbitrary term $d_pA$ of $y$ does not appear in list $L$.* The modified expression $y$ is called the corrected trial solution.

*The number $s$ of repeats for initial term $d_pA$ equals the multiplicity of the root $r$ which created atom $A$ in list $L$. 