Calculus III 2210-4
Sample Midterm Exam 3
Exam Date: 2 December 2005

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (ch15) Complete two of the following.
   (a) Find the directional derivative of \( f(x, y, z) = x^3y - y^2z^2 \) at \((-2, 1, 3)\) in the direction of \( \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \). See 15.5-7.
   (b) Let \( w = f(x, y, z) \) and \( x = r - s, y = s - t, z = t - r \). Derive from the chain rule that \( \frac{\partial}{\partial r} w + \frac{\partial}{\partial s} w + \frac{\partial}{\partial t} w = 0 \). See 15.6-30.
   (c) Find all points on the surface \( z = x^2 - 2xy - y^2 - 8x + 4y \) where the tangent plane is parallel to the \( xy \)-plane. See 15.7-13.
   (d) Find a point \((x_0, y_0, z_0)\) where surfaces \( z = x^2y \) and \( y = \frac{1}{4}x^2 + \frac{3}{4} \) intersect and such that the tangent planes at \((x_0, y_0, z_0)\) are perpendicular. See 15.7-16.

2. (Maxima and Minima) Complete two of the following.
   (a) State the critical point theorem and the second partials test. See 15.8.
   (b) Find the global maximum of \( f(x, y) = 3x + 4y \) on the rectangle \( 0 \leq x \leq 1, |y| \leq 1 \). See 15.8-11.
   (c) Find the minimum distance between the two lines with vector equations \( \mathbf{r}_1(t) = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \) and \( \mathbf{r}_2(t) = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \end{pmatrix} \). See 15.8-24.
   (d) Find the maximum of \( f(x, y) = xy \) subject to \( 4x^2 + 9y^2 = 36 \). See 15.9-2 or convert it to a one-variable max problem by solving for \( y \) in terms of \( x \).

3. (Double Integrals) Complete two of the following.
   (a) Define the double integral of \( f(x, y) \) over a rectangle \( Q \). See 16.1.
   (b) Let \( f(x, y) = 1 \) on \( R_1 \), \( f(x, y) = 3 \) on \( R_2 \), \( f(x, y) = 5 \) on \( R_3 \) and let \( R \) be the union of the three rectangles \( R_1, R_2, R_3 \). Suppose each rectangle has area 4. Find \( \int_R f(x, y) \, dA \).
   (c) Let \( R \) be defined by \( 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/2 \). Evaluate \( \int_R \sin(x + y) \, dA \).
   (d) Evaluate \( \int_0^3 \int_0^1 \frac{8x}{(x^2 + y + 2 + 1)^2} \, dy \, dx \). See 16.2-32.
   (e) Find the volume of the solid in the first octant bounded by the surface \( 9z = 36 - 9x^2 - 4y^2 \) and the coordinate planes. See 16.3-29.
   (f) Let \( R \) be the region in quadrant one bounded by \( x^2 + y^2 = 4 \) and the lines \( y = 0 \) and \( y = x \). Evaluate \( \int_R f(x, y) \, dA \) using polar coordinates, given \( f(x, y) = 1/(4 + x^2 + y^2) \). See 16.4-13.
4. (Surface Area) Complete two of the following.
   (a) Find the area of the part of the surface \( z = \sqrt{4-y^2} \) directly above the \( xy \)-plane square \( 1 \leq x \leq 2, 0 \leq y \leq 1 \). See 16.6-3.
   (b) Find the area of the part of the paraboloid \( z = x^2 + y^2 \) that is cut off by the plane \( z = 4 \). See 16.6-6 and example 2 page 712.
   (c) Find the center of mass of the homogeneous sphere \( x^2 + y^2 + z^2 = a^2 \) bewteen the planes \( z = a/2 \) and \( z = a/4 \) (\( a > 0 \) assumed). See 16.6-15.

5. (Triple Integrals) Complete two of the following.
   (a) Evaluate \( \int_0^2 \int_1^z \int_0^{\sqrt{x/z}} 2xyz \, dy \, dx \, dz \). See 16.7-5.
   (b) Evaluate \( \int \int \int_S dx \, dy \, dz \) where \( S \) is the tetrahedron with vertices \((0,0,0)\), \((3,2,0)\), \((0,3,0)\), \((0,0,2)\). See 16.7-15.
   (c) Find the volume of the solid in the first octant bounded by the 3D surfaces \( x^2 = y \) and \( z^2 = y \) and \( y = 1 \). See 16.7-21.
   (d) Find the center of mass of the homogeneous solid bounded above by \( z = 12 - 2x^2 - 2y^2 \) and below by \( z = x^2 + y^2 \). See 16.8-5.