Calculus III 2210-4
Sample Midterm Exam 3
Exam Date: 2 December 2005

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (ch15) Complete two of the following.
(a) Find the directional derivative of $f(x, y, z) = x^3y - y^2z^2$ at $(-2, 1, 3)$ in the direction of $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$. See 15.5-7.
(b) Let $w = f(x, y, z)$ and $x = r - s$, $y = s - t$, $z = t - r$. Derive from the chain rule that $\partial_r w + \partial_s w + \partial_t w = 0$. See 15.6-30.
(c) Find all points on the surface $z = x^2 - 2xy - y^2 - 8x + 4y$ where the tangent plane is parallel to the xy-plane. See 15.7-13.
(d) Find a point (x_0, y_0, z_0) where surfaces $z = x^2y$ and $y = \frac{1}{4}x^2 + \frac{3}{4}$ intersect and such that the tangent planes at (x_0, y_0, z_0) are perpendicular. See 15.7-16.

2. (Maxima and Minima) Complete two of the following.
(a) State the critical point theorem and the second partials test. See 15.8.
(b) Find the global maximium of $f(x, y) = 3x + 4y$ on the rectangle $0 \le x \le 1$, $|y| \le 1$. See 15.8-11.
(c) Find the minimum distance between the two lines with vector equations $\mathbf{r}_1(t) = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\mathbf{r}_2(t) = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$. See 15.8-24.
(d) Find the maximum of $f(x, y) = xy$ subject to $4x^2 + 9y^2 = 36$. See 15.9-2 or convert it to a one-variable max problem by solving for y in terms of x.

3. (Double Integrals) Complete two of the following.
(a) Define the double integral of $f(x, y)$ over a rectangle Q. See 16.1.
(b) Let $f(x, y) = 1$ on R_1, $f(x, y) = 3$ on R_2, $f(x, y) = 5$ on R_3 and let R be the union of the three rectangles R_1, R_2, R_3. Suppose each rectangle has area 4. Find $\int\int_R f(x, y)dA$.
(c) Let R be defined by $0 \le x \le \pi/2$, $0 \le y \le \pi/2$. Evaluate $\int\int_R \sin(x + y)dA$.
(d) Evaluate $\int_0^{\sqrt{3}} \int_0^1 \dfrac{8x}{(x^2 + y + 2 + 1)^2}\, dydx$. See 16.2-32.
(e) Find the volume of the solid in the first octant bounded by the surface $9z = 36 - 9x^2 - 4y^2$ and the coordinate planes. See 16.3-29.
(f) Let R be the region in quadrant one bounded by $x^2+y^2 = 4$ and the lines $y = 0$ and $y = x$. Evaluate $\int\int_R f(x, y)dA$ using polar coordinates, given $f(x, y) = 1/(4+x^2+y^2)$. See 16.4-13.

4. (Surface Area) Complete two of the following.

(a) Find the area of the part of the surface $z = \sqrt{4 - y^2}$ directly above the xy-plane square $1 \leq x \leq 2$, $0 \leq y \leq 1$. See 16.6-3.

(b) Find the area of the part of the paraboloid $z = x^2 + y^2$ that is cut off by the plane $z = 4$. See 16.6-6 and example 2 page 712.

(c) Find the center of mass of the homogeneous sphere $x^2 + y^2 + z^2 = a^2$ bewteen the planes $z = a/2$ and $z = a/4$ ($a > 0$ assumed). See 16.6-15.

5. (Triple Integrals) Complete two of the following.

(a) Evaluate $\int_0^2 \int_1^z \int_0^{\sqrt{x/z}} 2xyz dy dx dz$. See 16.7-5.

(b) Evaluate $\int\int\int_S dxdydz$ where S is the tetrahedron with vertices $(0, 0, 0)$, $(3, 2, 0)$, $(0, 3, 0)$, $(0, 0, 2)$. See 16.7-15.

(c) Find the volume of the solid in the first octant bounded by the 3D surfaces $x^2 = y$ and $z^2 = y$ and $y = 1$. See 16.7-21.

(d) Find the center of mass of the homogeneous solid bounded above by $z = 12 - 2x^2 - 2y^2$ and below by $z = x^2 + y^2$. See 16.8-5.