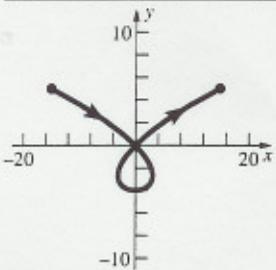


9. a.

$t$	$x$	$y$
-3	-15	5
-2	0	0
-1	3	-3
0	0	-4
1	-3	-3
2	0	0
3	15	5



b. Not simple; not closed

c.  $x^2 = t^6 - 8t^4 + 16t^2$

$t^2 = y + 4$

$x^2 = (y+4)^3 - 8(y+4)^2 + 16(y+4)$

$x^2 = y^3 + 4y^2$

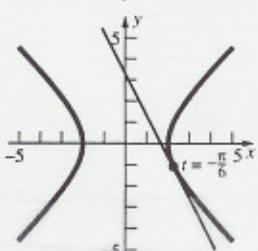
33.  $\frac{dx}{dt} = 2 \sec t \tan t, \frac{dy}{dt} = 2 \sec^2 t$

$\frac{dy}{dx} = \csc t$

At  $t = -\frac{\pi}{6}$ ,  $x = \frac{4}{\sqrt{3}}$ ,  $y = -\frac{2}{\sqrt{3}}$ , and  $\frac{dy}{dx} = -2$ .

Tangent line:  $y + \frac{2}{\sqrt{3}} = -2 \left( x - \frac{4}{\sqrt{3}} \right)$  or

$2\sqrt{3}x + \sqrt{3}y - 6 = 0$



41.  $\frac{dx}{dt} = 2e^t, \frac{dy}{dt} = \frac{9}{2}e^{3t/2}$

$L = \int_{\ln 3}^{2 \ln 3} \sqrt{4e^{2t} - \frac{81}{4}e^{3t}} dt = \int_{\ln 3}^{2 \ln 3} e^t \sqrt{4 - \frac{81}{4}e^t} dt$

$= \left[ -\frac{8}{243} \left( 4 - \frac{81}{4}e^t \right)^{3/2} \right]_{\ln 3}^{2 \ln 3}$

$= \frac{713\sqrt{713} - 227\sqrt{227}}{243}$

55.  $dx = dt$ ; when  $x = 0, t = -1$ ; when  $x = 1, t = 0$ .

$$\begin{aligned} \int_0^1 (x^2 - 4y) dx &= \int_{-1}^0 [(t+1)^2 - 4(t^3 + 4)] dt \\ &= \int_{-1}^0 (-4t^3 + t^2 + 2t - 15) dt \\ &= \left[ -t^4 + \frac{1}{3}t^3 + t^2 - 15t \right]_{-1}^0 = -\frac{44}{3} \end{aligned}$$

62. From Problem 61,

$x = (a-b)\cos t + b \cos\left(\frac{a-b}{b}t\right)$  and

$y = (a-b)\sin t - b \sin\left(\frac{a-b}{b}t\right)$ .

Substitute  $b = \frac{a}{4}$ .

$$\begin{aligned} x &= \left(\frac{3a}{4}\right)\cos t + \left(\frac{a}{4}\right)\cos(3t) \\ &= \left(\frac{3a}{4}\right)\cos t + \left(\frac{a}{4}\right)\cos(2t+t) \\ &= \left(\frac{3a}{4}\right)\cos t + \left(\frac{a}{4}\right)(\cos 2t \cos t - \sin 2t \sin t) \\ &= \left(\frac{3a}{4}\right)\cos t + \left(\frac{a}{4}\right)(\cos^3 t - \sin^2 t \cos t - 2\sin^2 t \cos t) \\ &= \left(\frac{3a}{4}\right)\cos t + \left(\frac{a}{4}\right)\cos^3 t - \left(\frac{3a}{4}\right)\cos t \sin^2 t \\ &= \left(\frac{3a}{4}\right)(\cos t)(1 - \sin^2 t) + \left(\frac{a}{4}\right)\cos^3 t = a \cos^3 t \end{aligned}$$

$y = \left(\frac{3a}{4}\right)\sin t - \left(\frac{a}{4}\right)\sin(3t)$

$= \left(\frac{3a}{4}\right)\sin t - \left(\frac{a}{4}\right)\sin(2t+t)$

$= \left(\frac{3a}{4}\right)\sin t - \left(\frac{a}{4}\right)(\sin 2t \cos t + \cos 2t \sin t)$

$= \left(\frac{3a}{4}\right)\sin t - \left(\frac{a}{4}\right)(2 \sin t \cos^2 t + \cos^2 t \sin t - \sin^3 t)$

$= \left(\frac{3a}{4}\right)\sin t - \left(\frac{3a}{4}\right)\sin t \cos^2 t + \left(\frac{a}{4}\right)\sin^3 t$