MATH 1030-008: EXAM 2 REVIEW

Originally, I was having you all memorize the basic compound interest formula. I now want you to memorize the general compound interest formula. This formula, when \( n = 1 \), is the same as the basic compound interest formula. Because they will not put it on your final, I want you to memorize it now. I would also like you to know the formulas for the general equation for a line, the change in dependent variable, and the rate of change. These are all as follows.

Memorize the following:

Compound interest formula for interest paid \( n \) times a year:

\[
A = P \left( 1 + \frac{APR}{n} \right)^{nY}
\]

General Equation for a line:

\[
y = mx + b, \text{ or}
\]

dependent variable = rate of change \( \times \) independent variable + initial value (or \( y \)-intercept).

Change in dependent variable:

\[
\text{change in dependent variable} = \text{rate of change} \times \text{change in independent variable}
\]

Rate of change: rate of change = slope = \( m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} \)

Know when and be able to use the following: (you don’t need to memorize these)

Compound interest formula for continuous compounding:

\[
A = P \times e^{(APR \times Y)}
\]

Savings plan formula:

\[
A = PMT \times \left[ \frac{\left(1 + \frac{APR}{n}\right)^{(nY)} - 1}{\frac{APR}{n}} \right]
\]

Loan payment formula:

\[
PMT = \frac{P \times \frac{APR}{n}}{1 - \left(1 + \frac{APR}{n}\right)^{-nY}}
\]

Doubling Time and Halving Time formulas:

- new value = initial value \( \times 2^{t/T_{\text{double}}} \)

\( T_{\text{double}} \approx \frac{70}{P} \)

- new value = initial value \( \times \left(\frac{1}{2}\right)^{t/T_{\text{half}}} \)

\( T_{\text{half}} \approx \frac{70}{P} \)

Logistic growth:

logistic growth rate = \( r \times \left( 1 - \frac{\text{population}}{\text{carrying capacity}} \right) \)

Equations for Exponential functions

\[
\begin{align*}
Q &= Q_0 \times (1 + r)^t \\
Q &= Q_0 \times 2^{t/T_{\text{double}}} \\
Q &= Q_0 \times \left(\frac{1}{2}\right)^{t/T_{\text{double}}} \\
\end{align*}
\]
1. Suppose you put a single deposit of $6,500 into an account with an APR of 5%, compounded daily. How much money will this be in 7 years?

2. Find the savings plan balance after 18 months with an APR of 6% and monthly payments of $300.

3. You put $200 per month in an investment plan that pays an APR of 4.5%. How much money will you have after 18 years? Compare this amount to the total deposits made over the time period.

4. You intend to create a college fund for your baby. If you can get an APR of 7.5% and want the fund to have a value of $150,000 after 18 years, how much should you deposit monthly?

5. Suppose you are 30 years old and would like to retire at age 60. Furthermore, you would like to have a retirement fund from which you can draw an income of $100,000 per year forever! How can you do it? Assume a constant APR of 6%.

6. Assume you have a balance of $5,000 on your credit card. The credit card APR is 21% and you want to pay off the balance in 3 years. Calculate the monthly payment and total payment under these conditions. Assume you make no additional charges on your card.

7. You have a car loan of $7,500 at a fixed APR of 5.8% for 3 years. Calculate your monthly payments and also calculate the total amount that you paid in interest over 3 years.
8. An account has an APR of 5.3%. You deposit $5,000. What is your APY when compounding occurs quarterly? monthly? daily?

9. Determine whether the following examples are exponential or linear growth (or decay).
   (a) The population of Tucson is increasing at a rate of 3% per year.
   
   (b) The price of a gallon of gasoline is increasing by $0.04 per week.
   
   (c) The value of your car is decreasing at a rate of 12% per year.

10. Oil consumption is increasing at a rate of 2.1% per year. What is its doubling time? By what factor will oil consumption increase in ten years?

11. Suppose there are 3 bacteria in a bottle at 5 pm and that the doubling time for bacteria is 3 minutes. How many bacteria will there be in the bottle at 5:09 pm?

12. The half-life of a drug in the bloodstream is 4 hours. By what factor does the concentration decrease in 24 hours?

13. Poaching is causing a population of elephants to decline by 9% per year. What is the half-life for the population? If there are 10,000 elephants today, how many will remain in 50 years?
14. Use the current annual growth rate of the United States, which is about 0.7%, to find the approximate doubling time of world population and to predict world population in 2050 based on a 2000 population of 6.0 billion.

15. One morning, there were 5 inches of snow on the ground. Then the winter storm started and snow started accumulating at a constant rate of 3 inches every 2 hours.

(a) Identify the independent and dependent variables:

independent variable ________________________  dependent variable ________________________

(b) Write a linear equation that describes this situation. ________________________

(c) How long did it take for the height of the snow to reach 17 inches? ________________________

16. For each of the following functions, find the slope of the graph and the y-intercept. Then sketch the graph for values of x between -10 and 10.

(a) \( y = -3x + 3 \)

(b) \( y = 4x + 1 \)

(c) \( y = 5 \)

(d) \( y = 2x \)

17. Suppose your pet dog weighed 2.5 pounds at birth and weighed 15 pounds after one year. Based on these two data points, find a linear function to predict your dog’s weight at 5 and 10 years of age. Comment on the validity of the model.
18. In 1993, Americans recycled 43.8 million tons of solid waste. In 1997, the figure grew to about 60.8 million tons. Find a linear function that fits the data. Use this function to predict the amount recycled in 2010. (Hint: Assume 1993 is time $t = 0$.

19. The half-life of aspirin in your bloodstream is 3.1 hours. What percent does it decrease by per hour (i.e., what is the rate of decay)?

20. Using the information in exercise 18, answer the following question. If you take 2 aspirin (650 mg total) at 8 o’clock in the morning, at what time should you take a second dose? (Assume that you take the second dose when your headache returns - when there is only 100 mg left in your bloodstream.)