INSTRUCTIONS: You will have 15 minutes to complete this quiz. No calculators, notes, books, neighbors, etc. are allowed. Show your work, and place your final answers in the rectangles provided.

(1) (3 pts) Let \( g(x) \) be a function with \( g'(2) = 0 \) and \( g''(2) = 4 \). What does the Second Derivative Test tell us about \( g \) at 2?

\[ g \text{ has a local minimum at } 2. \]

(2) \( f(x) = 3x^5 - 5x^3 + 1 \)

(a) (4 pts) Find all critical points of \( f \) on the interval \((-\infty, \infty)\). Make sure to include the \( x \) and \( y \) values of these points.

\[ f'(x) = 15x^4 - 15x^2 \quad \text{set} \quad 0 \]

divide by \( 15 \)

\[ x^4 - x^2 = 0 \]

\[ x^2(x^2 - 1) = 0 \]

\[ x^2(x+1)(x-1) = 0 \]

\[ x = 0, -1, 1 \]

\[ f(0) = 1 \]

\[ f(-1) = -3 + 5 + 1 = 3 \]

\[ f(1) = 3 - 5 + 1 = -1 \]
For reference: \( f(x) = 3x^5 - 5x^3 + 1 \)

(b) (4 pts) Where is \( f \) increasing and where is it decreasing? (That is, for what values of \( x \)?)

\[
f'(x) = 15x^2(x+1)(x-1)
\]

Increasing:

\((-\infty, -1] \) and \([1, \infty)\)

Decreasing:

\([-1, 1]\)

(c) (4 pts) Where is \( f \) concave up and where is it concave down? (Again, for what values of \( x \)?)

\[
f'(x) = 15x^4 - 15x^2
\]

\[
f''(x) = 60x^3 - 30x = 0
\]

divide by 30

\( 2x^3 - x = 0 \)

\( x(2x^2 - 1) = 0 \)

\( x = 0 \) or \( x = \pm \frac{1}{\sqrt{2}} \)

Concave Up:

\((-\frac{1}{\sqrt{2}}, 0) \) and \( (\frac{1}{\sqrt{2}}, \infty) \)

Concave Down:

\((-\infty, -\frac{1}{\sqrt{2}}) \) and \( (0, \frac{1}{\sqrt{2}}) \)
For reference: \( f(x) = 3x^5 - 5x^3 + 1 \)

(d) \( (4 \text{ pts}) \) List the \( x \)-values where \( f \) has local minima, local maxima, and inflection points.

- Local minima where \( x = 1 \)
- Local maxima where \( x = -1 \)
- Inflection points where \( x = -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \)

(e) \( (4 \text{ pts}) \) Sketch a graph of \( f \) using what you've found in parts (a) through (d). Make sure to label the scale on both axes (that is, where is “1”). Please mark inflection points with a dot, even if you are guessing at the \( y \)-values.