Midterm 2 Review

The exam will cover sections 2.4 - 2.9 and 3.1 - 3.5, with a little bit from section 3.6 (see below).

Please remember that SHOWING YOUR WORK IS IMPORTANT!! I want to see your complete process for each problem, written clearly for me to follow. The more points a problem is worth, the more work I want to see! And if you are stuck on a problem, at least tell me what you know about it, maybe even why you are stuck. I love giving partial credit when you show me that you’ve learned something!

As on the first exam, most questions will look very similar to problems we’ve done in class or on homework, but some will require you to apply your understanding in a new way. I suggest looking at all of the homework problems— even the ones that are in italics. For more practice, do even problems in sections where I assigned the odds, for example. Remember that you can check a lot of your answers at WolframAlpha.com.

Terms and Definitions to Know

- Right triangle definitions of trig functions (for example, $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$)
- Continuous Function: In practice on this exam, you should be able to recognize whether a specific function is continuous on some interval.
- Secant Line (and its slope)
- Average Rate of Change (i.e. Average Velocity)
- Tangent Line
- Instantaneous Rate of Change (i.e. Instantaneous Velocity)
- Derivative of $f$ at $c = f'(c) =$ slope of tangent line to $f$ at $c =$ instantaneous rate of change of $f$ at $c$
  
  Other notations: $f'(x) = \frac{d}{dx}f(x) = D_x f(x) = \frac{dy}{dx}$
- Second derivative ($f''(x)$), Third derivative ($f'''(x)$), Fourth derivative ($f^{(4)}(x)$), etc.
- Maximum/Minimum (plural: maxima/minima); Together, these are called extreme values
- Global max/min (also called absolute max/min): Actual highest/lowest point in some specified domain.
- Local max/min: Any “peak” or “valley”, even if it isn’t the highest or lowest.
• Critical points:
  1) Stationary points: points where \( f'(c) = 0 \)
  2) Endpoints: these are specific to a problem— the endpoints of a [closed] interval you are considering
  3) Singular points: points where \( f'(c) \) does not exist, or is undefined

• Concavity: concave up/concave down

• Sign line (for \( f' \) or \( f'' \))

• Inflection point: A point where a function changes concavity (switches from concave up to concave down or vice versa)

• Smooth function: If \( f'(x) \) is continuous, then \( f \) is smooth; its graph has no sharp points.

• Implicit differentiation

• Linear Approximation (approximate a function near some \( x = a \) with its tangent line there)

• Absolute error/Relative error

**Theorems and Such to Know**

• Calculating Derivatives:
  - \( \frac{d}{dx}(k) = 0 \)
  - **Power Rule**: \( \frac{d}{dx}(x^n) = nx^{n-1} \) for \( n \) any rational number (positive or negative, integer or fraction)
    
    Special Case of the Power Rule: \( \frac{d}{dx}(kx) = k \)
  - \( \frac{d}{dx}(kf(x)) = k \frac{d}{dx}f(x) \)
  - \( \frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x) \)
  - **Product Rule**: “The derivative of the front times the back plus the derivative of the back times the front.”
    \[
    \frac{d}{dx}(f(x) \cdot g(x)) = \left( \frac{d}{dx}f(x) \right) g(x) + \left( \frac{d}{dx}g(x) \right) f(x)
    \]
  - **Quotient Rule**: “Lo d-hi minus hi d-lo, square the bottom and off we go!!!!!!!!!”
    \[
    \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{(g(x))^2}
    \]
  - **Chain Rule**: “Derivative of the outside times the derivative of the guts”
\[ \frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) \]

- Derivatives of Trig Functions:
  - \( \frac{d}{dx} \sin x = \cos x \)
  - \( \frac{d}{dx} \cos x = -\sin x \)
  - \( \frac{d}{dx} \tan x = \sec^2 x \)
  - \( \frac{d}{dx} \sec x = \sec x \tan x \)
  - \( \frac{d}{dx} \cot x = -\csc^2 x \)
  - \( \frac{d}{dx} \csc x = -\csc x \cot x \)

- **Theorem:** If \( f \) is continuous on a closed interval \([a, b]\), then \( f \) attains both a maximum and minimum value on \([a, b]\), and both occur at critical points.

- What derivatives tell us about a function and its graph:
  - First derivative: increasing/decreasing
    - \( f'(x) = 0 \): \( f \) has a local minimum, a local maximum, or a flat point (example: \( f(x) = x^3 \) at \( x = 0 \))
    - \( f'(x) > 0 \): \( f \) is increasing
    - \( f'(x) < 0 \): \( f \) is decreasing
  - Second derivative: concavity
    - \( f''(x) > 0 \): \( f \) is concave up (like part of a bowl)
    - \( f''(x) < 0 \): \( f \) is concave down (like part of a hill)

- First Derivative Test: \( f \) has a local maximum at \( c \) if \( c \) is a critical point and \( f'(x) \) changes from positive to negative at \( c \). \( f \) has a local minimum at \( c \) if \( c \) is a critical point and \( f'(x) \) changes from negative to positive at \( c \).

- Second Derivative Test: If \( f'(c) = 0 \) and \( f''(c) > 0 \), then \( f \) has a local minimum at \( c \). If \( f'(c) = 0 \) and \( f''(c) < 0 \), then \( f \) has a local maximum at \( c \).
  - Note that this “test” tells you whether your function has a local min or max. It does not tell you concavity, even though concavity is also determined using \( f''(x) \).
  - Caution: if \( f''(c) = 0 \), the second derivative test is inconclusive—\( f \) might have a min, a max, or neither at \( c \).

- Approximation: \( f(x + \Delta x) \approx f(x) + f'(x) \Delta x \)

- Linear approximation of \( f \) at \( a \): \( L(x) = f(a) + f'(a)(x - a) \). (Remember that this is just the equation of the tangent line at \( x = a \))

- Absolute Error: \( \Delta y = f'(x) \Delta x \)
  
  This is the possible error in a calculation of \( y = f(x) \) if your measurement of \( x \) is within \( \Delta x \) of the actual value.

- Relative Error: \( \frac{\Delta y}{y} \)
• Mean Value Theorem: If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$, then there exists a value $c \in (a, b)$ where

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

• From section 3.6, I will only ask questions like: “Show whether or not the Mean Value Theorem can be applied to $f(x) = \sqrt{x}$ on the interval $[1, 3]$. If it can, find the value $c$ guaranteed by the theorem.”

(Some) Stuff I Might Ask You To Do

• Sketch the graph of a function and/or find: intervals where it’s increasing/decreasing, intervals where it’s concave up/down, local minima/maxima, absolute minimum/maximum, inflection points

• Optimization problems (section 3.4): Find the maximum or minimum... word problems.

• Identify whether a function attains a maximum/minimum on a given interval.

• Interpret a graph of $f'(x)$ (as in Quiz 5)

• Find the equation of a tangent line, possibly using implicit differentiation

  Could be worded as: Find the linear approximation to $f$ at $a$.

• Find a second derivative using implicit differentiation

• Related rates problems (section 2.8)

• Approximate a value without a calculator. For example, approximate $\sqrt{35}$.

• Find absolute error and relative error of some calculation.

• State the Mean Value Theorem

• Determine whether the Mean Value Theorem applies to a given function on a given interval, and if it does, find “$c$.”

• A problem like number 23 in section 3.6.