An Incomplete List of

Final Exam-type Questions

1) Sketch a graph of a function \( f \) with all of the following properties:
   (a) The domain of \( f \) is \([-5, 5]\).
   (b) \( f'(x) \) is defined on \((-5, -2), (-2, 1), \) and \((1, 5)\).
   (c) \( f \) is continuous everywhere except at \( x=1 \) in its domain.
   (d) \( \lim_{x \to 1^-} f(x) = f(1) \)
   (e) \( \lim_{x \to -4} f(x) = 0 \)
   (f) \( f'(0) = 0 \)

2) Evaluate: \( \lim_{x \to 5} \frac{\sin(x-5)}{x^2-25} \)

3) Suppose \( g(x) \) is continuous on \([-1, 3]\), and \( g(-1) = 5, \ g(3) = -\frac{1}{2} \). Do we know that \( g \) has a root in the interval \((-1, 3)\)? If so, by what theorem? If not, sketch a graph of such a function that does not have a root here.
4) Evaluate: \[ \int \sec x \tan x \, dx \]

5) Evaluate: \[ \int \sin x \cos x \, dx \]

6) Evaluate: \[ \int \sin y \cos^2 y \, dy \]

7) Approximate \( \sqrt{216} \) (using a derivative).

8) True or False? (True means \textit{always} true.)
   
   (a) If \( \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) \), then \( f \) is continuous at \( a \).
   
   (b) If \( f'(b) \) exists, then \( f \) is continuous at \( b \).
   
   (c) If \( g'(1) = 0 \), then \( g \) has either a local minimum or a local maximum at \( 1 \).

   (d) An inflection point of \( f \) is a point at which \( f''(x) = 0 \).

   (e) \( \int_a^b g(x) \, dx \leq \int_a^b f(x) \, dx \) implies that \( g(x) \leq f(x) \) on the interval \((a,b)\).

   (f) \( \int_a^b |h(x)| \, dx = |\int_a^b h(x) \, dx| \)
9) Find the volume of the solid obtained by rotating the shaded region below about:
(a) the x-axis
(b) the y-axis
(c) the line $x = -2$
(d) the line $y = -3$

10) Find the equation of the line tangent to the curve $y^2 + 2yx = x^3$ at the point $(1.6, 1)$. Hint: $(1.6)^2 \approx 2.6$
    $(1.6)^3 \approx 4.1$
    $(1.6)^4 \approx 6.5$

11) Given is the graph of $f(x)$. Sketch a graph of $f'(x)$ on the same axes.