MATH 2200 Final Exam

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Instructions

• Prove or disprove each of the “theorems” on the back of this sheet.

• You have 120 minutes for 10 problems, so roughly 12 minutes per problem.

• Each problem is worth 12 points.

• You don’t have to solve every problem to do well!

• Write your solutions on the blank paper provided. Don’t turn in this sheet!

• You don’t have to write the statements of the theorems, but label your solutions with the theorem numbers.

• Read the theorems carefully!

• Check cases of theorems and sketch proofs and counterexamples on scratch paper. Turn in only your final proofs and counterexamples.

• Write your name at the top of the first sheet of your written solutions. Staple the pages of your written solutions as you turn them in!
Problems

Theorem 1. Let $n \in \mathbb{Z}_{\geq 1}$. Then the sum of the first $n$ positive integers is $n(n + 1)$, namely

$$1 + 2 + \cdots + n = n(n + 1).$$

Theorem 2. Let $n \in \mathbb{Z}$. Then $n$ is odd if and only if $n^3$ is odd.

Theorem 3. Let $A$, $B$, and $C$ be sets, let $A \xrightarrow{f} B$ and $B \xrightarrow{g} C$ be functions, and let $A \xrightarrow{g \circ f} C$ be the composition of $f$ and $g$. If $g \circ f$ is injective, then $f$ is injective.

Theorem 4. Let $a, b, c \in \mathbb{Z}$ be nonzero integers. If $a \mid b$ and $b \mid c$, then $c \mid a$.

Theorem 5. Let $H_1$ and $H_2$ be connected subgraphs of a graph $G$. Then $H_1 \cap H_2$ is connected.

Theorem 6. Let $G$ be a bipartite graph. Then every walk in $G$ that starts and ends at the same vertex contains an even number of edges.

Theorem 7. Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{Z}_{\geq 1}$. If $ab \equiv 0 \pmod{n}$, then $a \equiv 0 \pmod{n}$ or $b \equiv 0 \pmod{n}$.

Theorem 8. Let $n \in \mathbb{Z}_{\geq 1}$. Then

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n + 1) = \frac{n(n + 1)(n + 2)}{3}.$$

Theorem 9. Let $H_1$ and $H_2$ be planar subgraphs of a graph $G$. Then $H_1 \cup H_2$ is planar.

Theorem 10. There are infinitely many primes.