Teaching Statement
Thomas Goller

Somewhere, somehow, mathematics education drifted away from creative problem solving and logical reasoning. In elementary school and high school, math courses train students to memorize techniques for solving those types of problems that will appear on the next exam. Many undergraduates are confused when I challenge them to think for themselves, and the question “Will this material be tested?” has become ubiquitous. Still, there is hope: some students develop an interest in mathematics despite the flaws of the education system; other students are flexible and a single course can change their lives. For me, teaching mathematics is about sharing the logic, creativity, and fun that have inspired me to become a mathematician. Math is a form of logical play; in a vibrant setting, students reveal themselves to be passionate players.

Since many undergraduates have experienced math as being dry and tedious, my most important task as an instructor is to grab their attention and never let go. Lecturing is a form of acting; in preparation for a lecture, I conserve energy so that I can release it at the chalkboard. I raise my voice, get excited about definitions like limit of a sequence and dimension of a vector space, and inject humor whenever possible. In my best lectures, I achieve a state of flow in which classroom boundaries dissolve, students are engaged and amused, and explanations emanate forth without a need for notes.

My goal in each lecture is to convey the big ideas as clearly as possible. Students need scaffolding to reach the summit of a course peak like the fundamental theorem of calculus. My job is to make the theorem compelling by providing motivation, examples, and intuition. In the first week of linear algebra, when the material is more level, I get excited about how algebra allows us to extend our geometric intuition for vectors in the plane and in space to higher dimensions. Excitement is contagious and helps students remember key moments in the classroom. Every course has themes and big ideas that can be used to energize an otherwise ordinary lecture.

I favor streamlined courses designed to get to the highlights as efficiently as possible. It is wasteful to spend half an hour on block matrices simply because they are “generally important”. Textbook authors tend to include too much material because of incentives to appeal to a broad audience, which distracts from the beautiful results and concepts. After teaching discrete math, I compiled my lecture notes into a book, which is available on my webpage and which I used when teaching the course a second time. The highlight of the chapter on number theory is the incredible RSA cryptosystem, which is preceded by a discussion of prime numbers, the Euclidean algorithm, modular arithmetic, congruences, and nothing else! Two other graduate student instructors, who based their courses on my book, found it to be invigorating.

In addition to lectures and the textbook, problems and exams are critical determinants of what students will learn from a course. Undergraduates expect homework to look like examples done in class, but it is essential that problems force students to think for themselves. In discrete math, some students “prove” a statement by filling in the gap between the assumptions and the conclusion with vaguely-relevant nonsense. My response was to rephrase every problem as “prove or disprove” and to craft tricky false statements; suddenly students were forced to think hard about the problem before attempting a solution. I heavily emphasized “give an example or state that no example exists” problems in foundations of analysis, which were so valuable that I resolved to consider them in every future course. In linear algebra, where students are less experienced, I used slightly simpler “invention” problems like “Invent a non-symmetric matrix whose row space equals its column space.” I love these problems because they test understanding of definitions and concepts while encouraging creativity. Many students had never taken a math course in which being creative was emphasized at all!
Over the years, I have discovered that the Undergraduate Colloquium is a great forum for testing out new ideas before springing them on students in a course. To find out how students would respond to abstract sweeping connections, I gave a talk “Theory of Everything” designed around an analogy between fields of math (categories) and languages, with functors being used to translate problems from one field to another. Chatting with a dozen students after the talk convinced me that such connections are extremely interesting to undergraduates, which inspired me to emphasize algebraic and geometric viewpoints in every linear algebra lecture. In the talk “Is humanity doomed?”, I experimented with heavy class participation by having students pair up and play iterated games of prisoner’s dilemma. The atmosphere in the room was so vibrant and students were so shocked by the reverse-induction proof that always defecting is an optimal strategy (contrary to their experience playing the game!) that I will seek out applications to game theory in any course I teach as a way to break up the usual routine of lectures.

Ultimately, interactions with students are the most meaningful part of teaching a course. I monitor every student’s progress by grading the tests and weekly quizzes before the next day of class, and I return graded work to each student individually to acknowledge his or her effort. I know I am achieving a good level of rapport with my students when they stay after class to chat about life or when they show up in force to office hours and problem sessions. When I taught linear algebra, I reserved the last three weeks of class for students to study an application of their choice, and it was amazing to watch students thrive in a creative environment and to have a chance to work with each student one-on-one. I met weekly throughout the semester with a student who asked me to help him understand the linear algebra in a paper on the Ford-Fulkerson algorithm for finding maximum flows in graphs. Since I had never heard of this algorithm, we both learned a lot over the course of many meetings. The most beautiful thing about teaching is the opportunity to interact with bright, highly-motivated students, who contribute their own ideas and knowledge.

My experiences as a graduate teaching instructor have culminated in a University Teaching Assistantship, which I am using to design a course on graph theory that I will teach this spring. Students will be heavily involved in class discussions, by giving lectures, and in individual research projects. I am thrilled to have the opportunity to interact so closely with just a dozen students and serve as a mentor to undergraduates pursuing research projects in mathematics and computer science. My dream when teaching a course is the creation of a friendly environment in which teacher and students work together as partners in a quest for logical enlightenment.