Quiz #1 Solutions

(1) Find the general solution of the linear system whose augmented matrix is $\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}$. **(4 points)**

Solution:

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix} \begin{matrix} \\ +2R_1 \end{matrix} \to \begin{bmatrix} 1 & -2 & -1 & 4 \\ 0 & 0 & -7 & 14 \end{bmatrix} \begin{matrix} \\ \cdot \frac{-1}{7} \end{matrix}$$
$$\to \begin{bmatrix} 1 & -2 & -1 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{matrix} +R_2 \\ \\ \end{matrix}$$
$$\to \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

The solution set is $\begin{cases} x_1 = 2 + 2x_2 \\ x_2 \text{ is free} \\ x_3 = -2 \end{cases}$.

(2) Invent a linear system of two equations in three variables that is inconsistent. **(2 points)**

Possible solution:

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

Possible solution:

$$\begin{aligned} 0x_1 + 6x_2 - x_3 &= 12 \\ 0x_1 + 0x_2 + 0x_3 &= 4 \end{aligned}$$

(3) Give a geometric description of Span $\left\{ \begin{bmatrix} 3 \\ -2 \end{bmatrix} \right\}$. **(1 point)**

Possible solution: The line in $\mathbb{R}^2$ passing through the origin and $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

Possible solution: The line in $\mathbb{R}^2$ through the origin with slope $-\frac{2}{3}$.

Possible solution: The line in the plane with equation $y = -\frac{2}{3}x$.

Description of my favorite solution: Sketch the line in the plane that the other solutions are describing in words.

(4) Bonus problem: Let A be a 6×4 matrix (6 rows, 4 columns) viewed as the augmented matrix of a linear system. Assume the linear system is consistent and that A is in reduced echelon form. Determine how many entries of A are equal to 0 in each of the following cases:

(a) A has as many entries equal to 0 as possible.

(b) A has as few entries equal to 0 as possible.

(1 bonus point)

Solution:

(a) 24 (A is the 6×4 matrix of all zeros)

(b) 18, for example

$$A = \begin{bmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{or} \quad A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$