# Answers to Odd-Numbered Exercises

## Chapter 1

### Section 1.1, page 10

**1.** The solution is $(x_1, x_2) = (-8, 3)$, or simply $(-8, 3)$.

**3.** $(2, 1)$

**5.** Replace Row 2 by its sum with -4 times Row 3, and then replace Row 1 by its sum with 3 times Row 3.

**7.** The solution set is empty.

**9.** $(16, 21, 14, 4)$ **11.** Inconsistent

**13.** $(5, 3, -1)$ **15.** Inconsistent

**17.** Calculations show that the system is inconsistent, so the three lines have no point in common.

**19.** $h \neq 2$ **21.** All $h$

**23.** Mark a statement True only if the statement is *always* true. Giving you the answers here would defeat the purpose of the true–false questions, which is to help you learn to read the text carefully. The *Study Guide* will tell you where to look for the answers, but you should not consult it until you have made an honest attempt to find the answers yourself.

**25.** $k - 2g + h = 0$

**27.** The row reduction of

$$\begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix} \text{ to } \begin{bmatrix} a & b & f \\ 0 & d - b(\frac{c}{a}) & g - f(\frac{c}{a}) \end{bmatrix}$$

shows that $d - b(\frac{c}{a})$ must be nonzero, since $f$ and $g$ are arbitrary. Otherwise, for some choices of $f$ and $g$ the second row could correspond to an equation of the form $0 = q$, where $q$ is nonzero. Thus $ad \neq bc$.

**29.** Swap Row 1 and Row 3; swap Row 1 and Row 3.

**31.** Replace Row 3 by Row 3 + (−4)Row 1; replace Row 3 by Row 3 + (4)Row 1.

**33.** Review Practice Problem 1 and then *write* a solution. The *Study Guide* has a solution.

### Section 1.2, page 21

**1.** Reduced echelon form: a and b. Echelon form: d. Not in echelon form: c.

**3.** $\begin{bmatrix} 1 & 2 & 0 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Pivot cols 1 and 3: $\begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$.

**5.** $\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \end{bmatrix}$, $\begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$

**7.** $\begin{cases} x_1 = -5 - 3x_2 \\ x_2 \text{ is free.} \\ x_3 = 3 \end{cases}$

**9.** $\begin{cases} x_1 = 3 + 2x_3 \\ x_2 = 3 + 2x_3 \\ x_3 \text{ is free.} \end{cases}$

**11.** $\begin{cases} x_1 = \frac{2}{3}x_2 - \frac{4}{3}x_3 \\ x_2 \text{ is free.} \\ x_3 \text{ is free.} \end{cases}$

**13.** $\begin{cases} x_1 = 5 + 3x_5 \\ x_2 = 1 + 4x_5 \\ x_3 \text{ is free.} \\ x_4 = 4 - 9x_5 \\ x_5 \text{ is free.} \end{cases}$

*Note:* The *Study Guide* discusses the common mistake $x_3 = 0$.

**15.** **a.** Consistent, with many solutions

**b.** Consistent, with many solutions

**17.** All $h$

**19.** **a.** Inconsistent when $h = 2$ and $k \neq 8$

**b.** Unique solution when $h \neq 2$

**c.** Many solutions when $h = 2$ and $k = 8$

**21.** Read the text carefully, and write your answers before you consult the *Study Guide*. Remember, a statement is true only if it is true in all cases.

**23.** Since there are four pivots (one in each column of the coefficient matrix), the augmented matrix must reduce to the form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & a \\ 0 & 1 & 0 & 0 & b \\ 0 & 0 & 1 & 0 & c \\ 0 & 0 & 0 & 1 & d \end{bmatrix}$$

and so

$$\begin{aligned} x_1 \qquad\qquad &= a \\ x_2 \qquad &= b \\ x_3 \quad &= c \\ x_4 &= d \end{aligned}$$

No matter what the values of $a$, $b$, $c$ and $d$, the solution exists and is unique.

**25.** If the coefficient matrix has a pivot position in every row, then there is a pivot position in the bottom row, and there is no room for a pivot in the augmented column. So, the system is consistent, by Theorem 2.

**27.** If a linear system is consistent, then the solution is unique if and only if *every column in the coefficient matrix is a pivot column; otherwise, there are infinitely many solutions*.

**29.** An underdetermined system always has more variables than equations. There cannot be more basic variables than there are equations, so there must be at least one free variable. Such a variable may be assigned infinitely many different values. If the system is consistent, each different value of a free variable will produce a different solution.

**31.** Yes, a system of linear equations with more equations than unknowns can be consistent. The following system has a solution ($x_1 = x_2 = 1$):

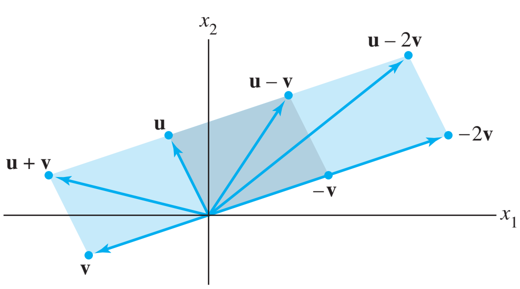
$$\begin{aligned} x_1 + x_2 &= 2 \\ x_1 - x_2 &= 0 \\ 3x_1 + 2x_2 &= 5 \end{aligned}$$

**33.** $p(t) = 1 + 3t + 2t^2$

## Section 1.3, page 32

**1.** $\begin{bmatrix} -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

**3.**




**5.** $x_1 \begin{bmatrix} 3 \\ -2 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 0 \\ -9 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}$,

$\begin{bmatrix} 3x_1 \\ -2x_1 \\ 8x_1 \end{bmatrix} + \begin{bmatrix} 5x_2 \\ 0 \\ -9x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}$,

$\begin{bmatrix} 3x_1 + 5x_2 \\ -2x_1 \\ 8x_1 - 9x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}$

$$\begin{aligned} 3x_1 + 5x_2 &= 2 \\ -2x_1 \quad\quad &= -3 \\ 8x_1 - 9x_2 &= 8 \end{aligned}$$

Usually the intermediate steps are not displayed.

**7.** $\mathbf{a} = \mathbf{u} - 2\mathbf{v}$, $\mathbf{b} = 2\mathbf{u} - 2\mathbf{v}$, $\mathbf{c} = 2\mathbf{u} - 3.5\mathbf{v}$, $\mathbf{d} = 3\mathbf{u} - 4\mathbf{v}$
Yes, every vector in $\mathbb{R}^2$ is a linear combination of **u** and **v**.

**9.** $x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

**11.** No, **b** is *not* a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

**13.** No, **b** is *not* a linear combination of the columns of $A$.

**15.** $h = 3$

**17.** Noninteger weights are acceptable, of course, but some simple choices are $0 \cdot \mathbf{v}_1 + 0 \cdot \mathbf{v}_2 = \mathbf{0}$, and

$1 \cdot \mathbf{v}_1 + 0 \cdot \mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$, $\quad 0 \cdot \mathbf{v}_1 + 1 \cdot \mathbf{v}_2 = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$

$1 \cdot \mathbf{v}_1 + 1 \cdot \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$, $\quad 1 \cdot \mathbf{v}_1 - 1 \cdot \mathbf{v}_2 = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$

**19.** Span $\{\mathbf{v}_1, \mathbf{v}_2\}$ is the set of points on the line through $\mathbf{v}_1$ and $\mathbf{0}$, because $\mathbf{v}_2$ is a multiple of $\mathbf{v}_1$.

**21.** *Hint:* Show that $\begin{bmatrix} 2 & 2 & h \\ -1 & 1 & k \end{bmatrix}$ is consistent for all $h$ and $k$. Explain what this calculation shows about Span $\{\mathbf{u}, \mathbf{v}\}$.

**23.** Before you consult your *Study Guide*, read the entire section carefully. Pay special attention to definitions and theorem statements, and note any remarks that precede or follow them.

**25.** **a.** No, three **b.** Yes, infinitely many
**c.** $\mathbf{a}_1 = 1 \cdot \mathbf{a}_1 + 0 \cdot \mathbf{a}_2 + 0 \cdot \mathbf{a}_3$

**27.** **a.** $5\mathbf{v}_1$ is the output of 5 days of operation of mine #1.
**b.** The total output is $x_1\mathbf{v}_1 + x_2\mathbf{v}_2$, so $x_1$ and $x_2$ should satisfy $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = \begin{bmatrix} 240 \\ 2824 \end{bmatrix}$.
**c.** **[M]** 1.73 days for mine #1 and 4.70 days for mine #2

**29.** $(17/14, -34/14, 16/14) = (17/14, -17/7, 8/7)$

**31.** **a.** $\begin{bmatrix} 10/3 \\ 2 \end{bmatrix}$
**b.** Add 3.5 g at (0, 1), add 0.5 g at (8, 1), and add 2 g at (2, 4).

**33.** Review Practice Problem 1 and then *write* a solution. The *Study Guide* has a solution.

## Section 1.4, page 40

**1.** The product is not defined because the number of columns (2) in the $3 \times 2$ matrix does not match the number of entries (3) in the vector.

**3.** **a.** $A\mathbf{x} = \begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = -2 \cdot \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} + 3 \cdot \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} -2 \\ 6 \\ -2 \end{bmatrix} + \begin{bmatrix} 6 \\ 3 \\ 18 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 16 \end{bmatrix}$

**b.** $A\mathbf{x} = \begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\cdot(-2)+2\cdot(3) \\ (-3)\cdot(-2)+1\cdot(3) \\ 1\cdot(-2)+6\cdot(3) \end{bmatrix}$
$= \begin{bmatrix} 4 \\ 9 \\ 16 \end{bmatrix}.$

Show your work here and for Exercises 4–6, but thereafter perform the calculations mentally.

**5.** $2\cdot\begin{bmatrix} 1 \\ -2 \end{bmatrix} - 1\cdot\begin{bmatrix} 2 \\ -3 \end{bmatrix} + 1\cdot\begin{bmatrix} -3 \\ 1 \end{bmatrix} - 1\cdot\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

**7.** $\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$

**9.** $x_1\begin{bmatrix} 5 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_3\begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$ and

$\begin{bmatrix} 5 & 1 & -3 \\ 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$

**11.** $\begin{bmatrix} 1 & 3 & -4 & -2 \\ 1 & 5 & 2 & 4 \\ -3 & -7 & 6 & 12 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 0 \end{bmatrix}$

**13.** Yes. (Justify your answer.)

**15.** The equation $A\mathbf{x} = \mathbf{b}$ is not consistent when $3b_1 + b_2$ is nonzero. (Show your work.) The set of $\mathbf{b}$ for which the equation *is* consistent is a line through the origin—the set of all points $(b_1, b_2)$ satisfying $b_2 = -3b_1$.

**17.** Only three rows contain a pivot position. The equation $A\mathbf{x} = \mathbf{b}$ does *not* have a solution for each $\mathbf{b}$ in $\mathbb{R}^4$, by Theorem 4.

**19.** The work in Exercise 17 shows that statement (d) in Theorem 4 is false. So all four statements in Theorem 4 are false. Thus, not all vectors in $\mathbb{R}^4$ can be written as a linear combination of the columns of $A$. Also, the columns of $A$ do *not* span $\mathbb{R}^4$.

**21.** The matrix $[\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3]$ does not have a pivot in each row, so the columns of the matrix do not span $\mathbb{R}^4$, by Theorem 4. That is, $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ does not span $\mathbb{R}^4$.

**23.** Read the text carefully and try to mark each exercise statement True or False before you consult the *Study Guide*. Several parts of Exercises 29 and 30 are *implications* of the form

"If ⟨statement 1⟩, then ⟨statement 2⟩"

or equivalently,

"⟨statement 2⟩, if ⟨statement 1⟩"

Mark such an implication as True if ⟨statement 2⟩ is true in all cases when ⟨statement 1⟩ is true.

**25.** $c_1 = -3, c_2 = -1, c_3 = 2$

**27.** The matrix equation can be written as $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + c_4\mathbf{v}_4 + c_5\mathbf{v}_5 = \mathbf{v}_6$, where $c_1 = -3$, $c_2 = 1, c_3 = 2, c_4 = -1, c_5 = 2$, and

$\mathbf{v}_1 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -4 \\ 1 \end{bmatrix},$

$\mathbf{v}_4 = \begin{bmatrix} 9 \\ -2 \end{bmatrix}, \quad \mathbf{v}_5 = \begin{bmatrix} 7 \\ -4 \end{bmatrix}, \quad \mathbf{v}_6 = \begin{bmatrix} 11 \\ -11 \end{bmatrix}$

**29.** *Hint:* Start with any $3 \times 3$ matrix $B$ in echelon form that has three pivot positions.

**31.** *Write* your solution before you check the *Study Guide*.

**33.** *Hint:* How many pivot columns does $A$ have? Why?

**35.** Suppose $\mathbf{y}$ and $\mathbf{z}$ satisfy $A\mathbf{y} = \mathbf{z}$. Then $5\mathbf{z} = 5A\mathbf{y}$. By Theorem 5(b), $5A\mathbf{y} = A(5\mathbf{y})$. So $5\mathbf{z} = A(5\mathbf{y})$, which shows that $5\mathbf{y}$ is a solution of $A\mathbf{x} = 5\mathbf{z}$. Thus the equation $A\mathbf{x} = 5\mathbf{z}$ is consistent.

**37.** **[M]** The columns do not span $\mathbb{R}^4$.

**39.** **[M]** The columns span $\mathbb{R}^4$.

**41.** **[M]** Delete column 4 of the matrix in Exercise 39. It is also possible to delete column 3 instead of column 4.

## Section 1.5, page 47

**1.** The system has a nontrivial solution because there is a free variable, $x_3$.

**3.** The system has a nontrivial solution because there is a free variable, $x_3$.

**5.** $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

**7.** $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3\begin{bmatrix} -9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 8 \\ -5 \\ 0 \\ 1 \end{bmatrix}$

**9.** $\mathbf{x} = x_2\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

**11.** *Hint:* The system derived from the *reduced* echelon form is

$$\begin{aligned} x_1 - 4x_2 \qquad + 5x_6 &= 0 \\ x_3 \qquad - x_6 &= 0 \\ x_5 - 4x_6 &= 0 \\ 0 &= 0 \end{aligned}$$

The basic variables are $x_1$, $x_3$, and $x_5$. The remaining variables are free. The *Study Guide* discusses two mistakes that are often made on this type of problem.

**13.** $\mathbf{x} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix} = \mathbf{p} + x_3\mathbf{q}$. Geometrically, the solution set is the line through $\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}$ parallel to $\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$.

**15.** Let $\mathbf{u} = \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \mathbf{p} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$. The solution of the homogeneous equation is $\mathbf{x} = x_2\mathbf{u} + x_3\mathbf{v}$, the plane through the origin spanned by $\mathbf{u}$ and $\mathbf{v}$. The solution set of the nonhomogeneous system is $\mathbf{x} = \mathbf{p} + x_2\mathbf{u} + x_3\mathbf{v}$, the plane through $\mathbf{p}$ parallel to the solution set of the homogeneous equation.

**17.** $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$. The solution set is the line through $\begin{bmatrix} 8 \\ -4 \\ 0 \end{bmatrix}$, parallel to the line that is the solution set of the homogeneous system in Exercise 5.

**19.** $\mathbf{x} = \mathbf{a} + t\mathbf{b}$, where $t$ represents a parameter, or

$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + t\begin{bmatrix} -5 \\ 3 \end{bmatrix}$, or $\begin{cases} x_1 = -2 - 5t \\ x_2 = 3t \end{cases}$

**21.** $\mathbf{x} = \mathbf{p} + t(\mathbf{q} - \mathbf{p}) = \begin{bmatrix} 3 \\ -3 \end{bmatrix} + t\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

**23.** It is important to read the text carefully and write your answers. After that, check the *Study Guide*, if necessary.

**25.** **a.** $A\mathbf{w} = A(\mathbf{p} + \mathbf{v}_h) = A\mathbf{p} + A\mathbf{v}_h = \mathbf{b} + \mathbf{0} = \mathbf{b}$

**b.** $A\mathbf{v}_h = A(\mathbf{w} - \mathbf{p}) = A\mathbf{w} - A\mathbf{p} = \mathbf{b} - \mathbf{b} = \mathbf{0}$

**27.** (*Geometric argument using Theorem 6*) Since the equation $A\mathbf{x} = \mathbf{b}$ is consistent, its solution set is obtained by translating the solution set of $A\mathbf{x} = \mathbf{0}$, by Theorem 6. So the solution set of $A\mathbf{x} = \mathbf{b}$ is a single vector if and only if the solution set of $A\mathbf{x} = \mathbf{0}$ is a single vector, and that happens if and only if $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

(*Proof using free variables*) If $A\mathbf{x} = \mathbf{b}$ has a solution, then the solution is unique if and only if there are no free variables in the corresponding system of equations, that is, if and only if every column of $A$ is a pivot column. This happens if and only if the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

**29.** **a.** When $A$ is a $4 \times 4$ matrix with three pivot positions, the equation $A\mathbf{x} = \mathbf{0}$ has a free variable and hence has nontrivial solutions.

**b.** With three pivot positions, $A$ does not have a pivot position in each of its four rows. By Theorem 4 in Section 1.4, the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for every possible $\mathbf{b}$. The word "possible" in the exercise means that the only vectors considered in this case are those in $\mathbb{R}^4$, because $A$ has four rows.

**31.** **a.** When $A$ is a $3 \times 2$ matrix with two pivot positions, each column is a pivot column. So the equation $A\mathbf{x} = \mathbf{0}$ has no free variables and hence no nontrivial solution.

**b.** With two pivot positions and three rows, $A$ cannot have a pivot in every row. So the equation $A\mathbf{x} = \mathbf{b}$ cannot have a solution for every possible $\mathbf{b}$ (in $\mathbb{R}^3$), by Theorem 4 in Section 1.4.

**33.** Your example should have the property that the sum of the entries in each row is zero. Why?

**35.** One answer: $\mathbf{x} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

**37.** One answer is $A = \begin{bmatrix} 1 & -4 \\ 1 & -4 \end{bmatrix}$. The *Study Guide* shows how to analyze the problem in order to construct $A$. If $\mathbf{b}$ is any vector *not* a multiple of the first column of $A$, then the solution set of $A\mathbf{x} = \mathbf{b}$ is empty and thus cannot be formed by translating the solution set of $A\mathbf{x} = \mathbf{0}$. This does not contradict Theorem 6, because that theorem applies when the equation $A\mathbf{x} = \mathbf{b}$ has a nonempty solution set.

**39.** Suppose $A\mathbf{v} = \mathbf{0}$ and $A\mathbf{w} = \mathbf{0}$. Then, since $A(\mathbf{v} + \mathbf{w}) = A\mathbf{v} + A\mathbf{w}$ by Theorem 5(a) in Section 1.4, $A(\mathbf{v} + \mathbf{w}) = A\mathbf{v} + A\mathbf{w} = \mathbf{0} + \mathbf{0} = \mathbf{0}$. Now, let $c$ and $d$ be scalars. Using both parts of Theorem 5, $A(c\mathbf{v} + d\mathbf{w}) = A(c\mathbf{v}) + A(d\mathbf{w}) = cA\mathbf{v} + dA\mathbf{w} = c\mathbf{0} + d\mathbf{0} = \mathbf{0}$.

## Section 1.6, page 54

**1.** The general solution is $p_G = .875p_S$, with $p_S$ free. One equilibrium solution is $p_S = 1000$ and $p_G = 875$. Using fractions, the general solution could be written $p_G = (7/8)p_S$, and a natural choice of prices might be $p_S = 80$ and $p_G = 70$. Only the *ratio* of the prices is important. The economic equilibrium is unaffected by a proportional change in prices.

**3.** **a.**

| | Distribution of Output from: | | | | |
|---|---|---|---|---|---|
| | F&P | Man. | Ser. | | |
| Output | ↓ | ↓ | ↓ | Input | Purchased by: |
| | .1 | .1 | .2 | → | F&P |
| | .8 | .1 | .4 | → | Man. |
| | .1 | .8 | .4 | → | Ser. |

**b.** $\begin{bmatrix} .9 & -.1 & -.2 & 0 \\ -.8 & .9 & -.4 & 0 \\ -.1 & -.8 & .6 & 0 \end{bmatrix}$

**c.** **[M]** $p_{F\&P} \approx 30$, $p_M \approx 71$, $p_S = 100$.

**5.** **a.**

| | Distribution of Output from: | | | | | |
|---|---|---|---|---|---|---|
| | Ag. | Man. | Ser. | Transp. | | |
| Output | ↓ | ↓ | ↓ | ↓ | Input | Purchased by: |
| | .20 | .35 | .10 | .20 | → | Ag. |
| | .20 | .10 | .20 | .30 | → | Man. |
| | .30 | .35 | .50 | .20 | → | Ser. |
| | .30 | .20 | .20 | .30 | → | Transp. |

**b.** One solution is $p_A = 7.99$, $p_M = 8.36$, $p_S = 14.65$, and $p_T = 10.00$.

**c.** Distribution of Output from:

| Output | Ag. ↓ | Man. ↓ | Ser. ↓ | Transp. ↓ | Input | Purchased by: |
|---|---|---|---|---|---|---|
| | .20 | .35 | .10 | .20 | → | Ag. |
| | .10 | .10 | .20 | .30 | → | Man. |
| | .40 | .35 | .50 | .20 | → | Ser. |
| | .30 | .20 | .20 | .30 | → | Transp. |

**d.** One solution is $p_A = 7.81$, $p_M = 7.67$, $p_S = 15.62$, and $p_T = 10.00$.
The campaign has benefited Services the most.

**7.** $3NaHCO_3 + H_3C_6H_5O_7 \to Na_3C_6H_5O_7 + 3H_2O + 3CO_2$

**9.** $B_2S_3 + 6H_2O \to 2H_3BO_3 + 3H_2S$

**11.** [**M**] $16MnS + 13As_2Cr_{10}O_{35} + 374H_2SO_4$
$\to 16HMnO_4 + 26AsH_3 + 130CrS_3O_{12} + 327H_2O$

**13. a.** $\begin{cases} x_1 = x_3 - 40 \\ x_2 = x_3 + 10 \\ x_3 \text{ is free} \\ x_4 = x_6 + 50 \\ x_5 = x_6 + 60 \\ x_6 \text{ is free} \end{cases}$ **b.** $\begin{cases} x_2 = 50 \\ x_3 = 40 \\ x_4 = 50 \\ x_5 = 60 \end{cases}$

**15.** $\begin{cases} x_1 = 60 + x_6 \\ x_2 = -10 + x_6 \\ x_3 = 90 + x_6 \\ x_4 = x_6 \\ x_5 = 80 + x_6 \\ x_6 \text{ is free} \end{cases}$
In order for the flow to be nonnegative, $x_6 \geq 10$

## Section 1.7, page 60

Justify your answers to Exercises 1–22.

**1.** Lin. indep. **3.** Lin. depen.

**5.** Lin. indep. **7.** Lin. depen.

**9. a.** No $h$ **b.** All $h$

**11.** $h = -4$ **13.** All $h$

**15.** Lin. depen. **17.** Lin. depen. **19.** Lin. indep.

**21.** If you consult your *Study Guide* before you make a good effort to answer the true-false questions, you will destroy most of their value.

**23.** $\begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

**25.** $\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

**27.** All four columns of the $6 \times 4$ matrix $A$ must be pivot columns. Otherwise, the equation $A\mathbf{x} = \mathbf{0}$ would have a free variable, in which case the columns of $A$ would be linearly dependent.

**29.** $A$: Any $3 \times 2$ matrix with the second column a multiple of the first will have the desired property.
$B$: Any $3 \times 2$ matrix with two nonzero columns such that neither column is a multiple of the other will work. In this case, the columns form a linearly independent set, and so the equation $B\mathbf{x} = \mathbf{0}$ has only the trivial solution.

**31.** $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

**33.** True, by Theorem 7. (The *Study Guide* adds another justification.)

**35.** True, by Theorem 9.

**37.** True. A linear dependence relation among $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ may be extended to a linear dependence relation among $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ by placing a zero weight on $\mathbf{v}_4$.

**39.** You should be able to work this important problem without help. *Write* your solution before you consult the *Study Guide*.

**41.** [**M**] Using the pivot columns of $A$,

$$B = \begin{bmatrix} 3 & -4 & 7 \\ -5 & -3 & -11 \\ 4 & 3 & 2 \\ 8 & -7 & 4 \end{bmatrix}$$

Other choices are possible.

**43.** [**M**] Each column of $A$ that is not a column of $B$ is in the set spanned by the columns of $B$.

## Section 1.8, page 68

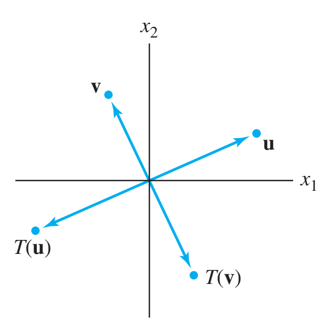
**1.** $\begin{bmatrix} 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 2a \\ 2b \end{bmatrix}$ **3.** $\mathbf{x} = \begin{bmatrix} 7 \\ 6 \\ 3 \end{bmatrix}$, unique solution

**5.** $\mathbf{x} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, not unique **7.** $a = 5, b = 6$

**9.** $\mathbf{x} = x_3 \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$

**11.** Yes, because the system represented by $[\,A \quad \mathbf{b}\,]$ is consistent.

**13.**




A reflection through the origin