All Quiz Problems

Quiz #1

(1) Find two different linear combinations of the three vectors \( \vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) and \( \vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \) and \( \vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \) that produce \( \vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \). (4 points)

(2) Find two different vectors \( \vec{v} \) and \( \vec{w} \) that are perpendicular to \( \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \) and to each other. (4 points)

(3) Let \( \vec{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \) and \( \vec{v} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} \) and \( \vec{w} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \). Compute the linear combination \( 4\vec{u} - 3\vec{v} + \vec{w} = \vec{b} \) and write it as a matrix-vector multiplication \( A\vec{x} = \vec{b} \). (4 points)

Quiz #2

(1) Compute the inverse of \( A \) using the Gauss-Jordan method starting with the block matrix \( [A \quad I] \). Show your work!

\[
A = \begin{bmatrix} 1 & 0 & 5 \\ 4 & 2 & 20 \\ 0 & 4 & -5 \end{bmatrix}
\] (8 points)

(2) Write the following problem in a 2 by 2 matrix form \( A\vec{x} = \vec{b} \):

"X is three times as old as Y and their ages add to 72."

You do not need to solve the problem! (4 points)

Quiz #3

(1) True or false (you don’t need to justify your answer):

   (a) If \( A \) and \( B \) are symmetric then \( AB \) is symmetric. (1 point)

   (b) The line in \( \mathbb{R}^2 \) with equation \( x + y = 0 \) is a subspace. (1 point)

   (c) The column space of \( \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ -5 & -10 \end{bmatrix} \) is a plane in \( \mathbb{R}^3 \). (1 point)
(d) \((AB)^T = A^TB^T\). (1 point)

(2) Let \(P\) be the plane in \(\mathbb{R}^3\) with equation \(2x - y + z = -2\). Find two vectors in \(P\) and check that their sum is not in \(P\). (This shows that \(P\) is not a subspace!) (4 points)

(3) Find the \(A = LU\) factorization for \(A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 3 \\ -1 & -2 & 4 \end{bmatrix}\). (4 points)

(4) Optional problem if you finish the other parts early: Find all \(2 \times 2\) symmetric matrices \(A\) with integer entries that satisfy \(A^2 = I\). (Hint: There are six.) (0 points)

Quiz #4

(1) Find bases and dimensions for the four subspaces associated with \(A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}\). (9 points)

(2) Show that the following vectors are dependent:

\[
\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.
\] (3 points)

(3) Optional problem: Let \(R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}\). Check that the columns of \(R\) are perpendicular unit vectors. Thinking of \(R\) as a function \(\mathbb{R}^2 \xrightarrow{R} \mathbb{R}^2\), how does \(R\) act on vectors in the plane? (If you’re stuck, first try to answer this question for \(\theta = \frac{\pi}{2}\) by computing \(R \begin{bmatrix} 1 \\ 0 \end{bmatrix}\) and \(R \begin{bmatrix} 0 \\ 1 \end{bmatrix}\).) How does \(\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}\) act on vectors in the plane, and how can you deduce this from your answer for \(R\)? (0 points)

Quiz #5

(1) Write down three equations for the line \(b = C + Dt\) to go through \(b = 1\) at \(t = -1\), \(b = -1\) at \(t = 0\), and \(b = 3\) at \(t = 1\). Find the least squares solution \(\hat{x} = \begin{bmatrix} C \\ D \end{bmatrix}\) and draw the closest line. (Hint: write your equations as \(A\vec{x} = \vec{b}\), which has no solution, so instead solve \(A^TA\vec{x} = A^T\vec{b}\) for the best approximation \(\hat{x}\).) (8 points)

(2) Find orthonormal vectors \(\vec{q}_1\) and \(\vec{q}_2\) in the plane spanned by \(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}\) and \(\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}\). (4 points)
(3) Optional problem: Draw a picture, write down a riddle, or tell me something you find interesting that I probably don’t know. (0 points)

Quiz #6

All three problems below are about the matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$.

(1) Find the eigenvalues $\lambda_1, \lambda_2$ and eigenvectors $\vec{x}_1, \vec{x}_2$ for $A$. (6 points)

(2) Check that $A\vec{x}_1 = \lambda_1 \vec{x}_1$ and $A\vec{x}_2 = \lambda_2 \vec{x}_2$. (2 points)

(3) Factor $A$ into $A = SAS^{-1}$. (4 points)

(4) Optional problem: Surprise me! (0 points)

Quiz #7

(1) If $C = F^{-1}AF$ and also $C = G^{-1}BG$, what matrix $M$ gives $B = M^{-1}AM$? (You must show your work to receive full credit!) (4 points)

(2) Suppose a linear $T$ transforms $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$. Find $T \begin{bmatrix} 5 \\ 3 \end{bmatrix}$. (You must show your work to receive full credit!) (4 points)

(3) For each of the following functions $\mathbb{R}^2 \xrightarrow{T} \mathbb{R}^2$, is $T$ linear or not linear? (You don’t have to show any work.) (4 points)

(a) $T \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.
(b) $T \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2v_1 \\ 2v_1 \end{bmatrix}$.
(c) $T \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \sin v_2 \\ v_1 \end{bmatrix}$.
(d) $T \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 - 1 \\ v_2 \end{bmatrix}$.

(4) (Optional fun art/geometry/creative writing problem) Consider the linear transformation $\mathbb{R}^2 \xrightarrow{T} \mathbb{R}^2$ defined by $T(\vec{v}) = A\vec{v}$, where $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$. ($T$ acts by a combination of vertical and horizontal stretching and horizontal shearing.) Invent a parallelogram in $\mathbb{R}^2$ that does not have the origin as one of its vertices, and check that the action of $T$ takes that parallelogram to another parallelogram. Sketch both parallelograms, give them names, and write a poem about them (“O parallel Peter, how perfect doth thine angles be; far sweeter than sharp-cornered Cassidy!”) (0 points)