Math 2200 Homework 7

Due Date: November 6

Please write neatly and leave enough space so I can write comments! Remember that there are solutions to all the odd-numbered exercises in the textbook. I will mainly be looking at your solutions to the even-numbered exercises from the textbook and all the exercises I write, so make sure you attempt most of those. For formal proofs, please state the theorem being proved, write “proof”, and then give your argument using complete English sentences. Please show your work on all problems!

Repeat from Homework 6

You only have to write two proofs, and you have already tried them once, so please fix your mistakes and write the most crisp and clear proofs you can!

Exercise 1. Use induction to prove the following summation formula:

**Theorem.** Let \( n \in \mathbb{Z}^{\geq 1} \). Then the sum of the first \( n \) even positive integers is \( n(n + 1) \).

Exercise 2. Use induction to prove the power rule for differentiating polynomials:

**Theorem.** For \( n \in \mathbb{Z}^{\geq 1} \), let \( f_n(x) = x^n \). Then \( f'_n(x) = nx^{n-1} \).

(Do not use the power rule in the proof – you are trying to prove it works! For the base case \( n = 1 \), you may simply assert that the derivative of \( x \) is \( 1 \); one proves this using the definition of the derivative, but you don’t have to write that out. For the inductive step, use the product rule to differentiate \( x^{k+1} = x^k \cdot x \).)

Section 6.1 (p. 396-7)

\# 1, \# 3, \# 5, \# 7, \# 8, \# 29, \# 34, \# 35, \# 36.