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Abstract

The thermal conductivity of sea ice plays a critical role in climate modeling by deq termining the coupling between the temperature fields in the ocean and atmosphere 10 through the sea ice layer. This study aims to find a composite material model that best 11 describes the heat transfer through the sea ice by solving an inverse problem. Sea ice 12 temperature data from the 2007 Sea Ice Mass Balance in the Antarctic (SIMBA) project 13 are divided into two sets, with one set used for model training and the other for valida-14 tion. This study proposes a layer-based linear model, as well as a nonlinear diffusivity 15 model. For each proposed model, an estimated temperature profile is recovered from the 16 sea ice heat equation and is then compared with the training data to calculate the root 17 mean squared error (RMSE). Optimal model parameters are obtained by minimizing 18 the RMSE via numerical approximations. It is found that a five-layer model and the 19 nonlinear model are the most effective. An extreme diffusivity value discovered in the 20 bottom layer of the model indicates that there may be convective processes contributing 21 to the heat transport. To account for this, the sea ice heat equation is generalized to 22 an effective advection-diffusion equation with a brine velocity field term to refine our 23 models. The diffusivity values of the new models are converted to conductivity values, 24 and we obtain good agreement between the model and the data. 25

²⁶ 1 Introduction

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The Arctic and Antarctic sea ice covers form a key component of Earth's climate system. The sea ice packs interact with the ocean below and the atmosphere above, as well as internally, within themselves via mechanical and thermal interactions. Modeling and predicting the evolution of the polar ice packs and their interactions with the ocean and atmosphere depends on accurate representations of dynamic and thermodynamic processes (Thomas

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