Figure 3. Comparison of real Arctic melt ponds with metastable equilibria in our melt pond long model. (a) Ising model simulation. (b) Real melt pond photo. Figure 3a courtesy of Donald Perovich. Vast labyrinthine ponds on the surface of melting Arctic sea ice are key players in the polar climate system and upper ocean ecology. Researchers have adapted the Ising model, which was originally developed to understand magnetic materials, to study the geometry of meltwater’s distribution over the sea ice surface. In an article on page 5, Kenneth Golden, Yiping Ma, Courtney Strong, and Ivan Sudakov explore model predictions.

By Yu Jin and Suzanne Lenhart

Flow regimes can change significantly over time and space and strongly impact all levels of river biodiversity, from the individual to the ecosystem. Invasive species in rivers—such as bighead and silver carp, as well as quagga and zebra mussels—continue to cause damage. Management of these species may include targeted adjustment of flow rates in rivers, based on recent research that examines the effects of river morphology and water flow on rivers’ ecological statuses. While many previous methodologies rely on habitat suitability models or oversimplification of the hydrodynamics, few studies have focused on the integration of ecological dynamics into water flow assessments.

Earlier work yielded a hybrid modeling approach that directly links river hydrology with stream population models [3]. The hybrid model’s hydrodynamic component is based on the water depth in a gradually varying river structure. The model derives the steady advective flow from this structure and relates it to flow features like water discharge, depth, velocity, cross-sectional area, bottom roughness, bottom slope, and gravitational acceleration. This approach facilitates both theoretical understanding and the generation of quantitative predictions, thus providing a way for scientists to analyze the effects of river fluctuations on population processes.

When a population spreads longitudinally in a one-dimensional (1D) river with spatial heterogeneities in habitat and temporal fluctuations in discharge, the resulting hydrodynamic population model is

\[
N(t) = -\frac{\lambda(t)}{\mu(t)} N(t) + \int_{t}^{t+\Delta t} \left( \frac{J(x, t)}{\mu(t)} \mathcal{R}(x, t) \right) dx - \int_{t}^{t+\Delta t} \frac{\lambda(t)}{\mu(t)} N(t) dx,
\]

\[
N(0, t) = 0 \quad \text{on} \quad (0, t), \quad \lambda(t) = \lambda(t) \quad \text{on} \quad (0, t) \quad \text{for} \quad t = 0
\]

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**Modeling Resource Demands and Constraints for COVID-19 Intervention Strategies**

By Erin C.S. Acquista, Walt Beasley, Pat Finley, Katherine Klise, Monear Malvankar, and Emma Stanislavskii

As the world desperately attempts to control the spread of COVID-19, the need for a model that accounts for realistic trade-offs between time, resources, and corresponding epidemiological implications is apparent. Some early mathematical models of the outbreak compared trade-offs for non-pharmaceutical interventions [3], while others derived the necessary level of test coverage for case-based interventions [4] and demonstrated the value of prioritized testing for close contacts [7]. Isolated analyses provide valuable insights, but real-world intervention strategies are interconnected. Contact tracing is the lynchpin of infection control [6] and forms the basis of prioritized testing. Therefore, quantifying the effectiveness of contact tracing is crucial to understanding the real-life implications of disease control strategies.

**Contact Tracing Demands**

Contact tracers are skilled, culturally competent interviewers who apply their knowledge of disease and risk factors when notifying people who have come into contact with COVID-19-infected individuals. They also continue to monitor the situation after case investigations [1].

Case investigation consists of four steps:
1. Identify and notify cases
2. Interview cases
3. Locate and notify contacts
4. Monitor contacts

Most health departments are implementing case investigation, contact identification, and quarantine to disrupt COVID-19 transmission. The timeliness of contact tracing is constrained by the length of the infectious period, the turn-around time for testing and result reporting, and the ability to successfully reach and interview patients and their contacts. The European Centre for Disease Prevention and Control approximates that contact tracers spend one to two hours conducting an interview [2].

Estimates regarding the timelines of other steps are limited to subject matter expert elicitation and can vary based on cases’ access to phone service or willingness to participate in interviews.

**Bounded Exponential**

The fundamental structure of our model follows traditional susceptible-exposed-infected-infected recovered (SEIR) compartmental modeling [5]. We add an asymptomatic population, a hospitalized population, and disease-related deaths D, as well as corresponding quarantine states. We define the states \( \{S, E, A, L, H, D, D_L, D_R\} \) for our compartments, such that \( i = 0 \) and \( j = 1 \) correspond to unquarantined and quarantined respectively. Rather than focus on the dynamics that are associated with the state transition diagram in Figure 1, we introduce a formulation for the real-time demands on contact tracers’ time as a function of infection prevalence, while also respecting constraints on resources.

When the work that is required to investigate new cases and monitor existing contacts exceeds available resources, backlog develops. To simulate this backlog, we introduce a new compartment \( C \) for tracking the dynamic states of:\n
\[
\frac{dC}{dt} = [\text{flow into}] - [\text{flow out}]
\]

Flow into the backlog compartment, represented by \( \text{flow in} \), reflects case identification that is associated with the following transitions in the model:

- The rate of random testing: \( q_1(t) A(t) \rightarrow A(t) \) and \( q_2(t) I(t) \rightarrow I(t) \)
- Testing triggered by contact tracing: \( q_3(t) A(t) \rightarrow A(t) \), \( q_4(t) I(t) \rightarrow I(t) \), and \( q_5(t) E(t) \rightarrow A(t) \)

- The population that was missed by the non-pharmaceutical interventions that require hospitalization: \( \tau_H(0, t) = 0 \rightarrow H(t) \)

Here, \( q_i(t) \) defines the time-dependent rate of random testing, \( q_i(t) \) signifies the time-dependent rate of testing that is triggered by contact tracing, and \( \tau_H(t) \) is the inverse of the expected amount of time for which an infected individual is symptomatic before hospitalization. These terms collectively provide the simulated number of newly-identified positive COVID-19 cases. However, we also need the average number of contacts per case. We thus define function \( K(t, \tau_H(t)) \) that depends on the average number of contacts a day \( k \), the average number of days for which an individual is infectious before going into isolation \( \tau_H(t) \), and the likelihood that the individual

See COVID-19 Intervention on page 3

Figure 1. Disease state diagram for the compartmental infectious disease model. Figure courtesy of the authors.
From Magnets to Melt Ponds

By Kenneth M. Golden, Yiping Ma, Courtesy Strong, and Ivan Sudakov

When the snow on top of Arctic sea ice begins to melt in late spring, small pools of water form on the surface. As the melt season progresses, these simple shaped meter-scale pools grow and coalesce into kilometer-scale tributaries of cenereal blue with complex, self-similar boundaries. The fractal dimension of these boundaries transitions from one to roughly two as the area increases through a critical regime that is centered around 100 square meters. While the white, snowy surface of the sea ice reflects most of the incident sunlight, the darker melt ponds act like windows and allow significant light to penetrate the ice and scatter underwater. Melt ponds thus help control the amount of solar energy that the ice pack and upper ocean absorb, strongly influencing ice melting rates and the ecology of the polar marine environment. They largely determine sea ice albedo—the ratio of reflected to incident sunlight—which is a key parameter in climate modeling.

When viewed from a helicopter, the beautiful patterns of dark and light on the surface of melting sea ice are reminiscent of structures that applied mathematicians sometimes see when studying phase transitions and coarsening processes in materials science. They also resemble the complex regions of aligned spins, or magnetic domains, that are visible in magnetic materials. Figure 1 compares two examples of magnetic domains with similar patterns that are formed by melt ponds on Arctic sea ice. Magnetic energy is lowered when nearby spins align with each other, which produces the domains. At higher temperatures, thermal fluctuations dominate the tendency of the domains’ magnetic moments to also align, with no net magnetization $M$ of the material unless one applies an external magnetic field $H$ to induce alignment. However, the tendency for overall alignment takes over at temperatures below the Curie point $T_C$, and the material remains magnetized even as the applied field $H$ vanishes, where the remaining non-zero magnetization ($M = 0$) is called spontaneous or residual.

The prototypical model of a magnetic material based on a lattice of interacting binary spins is the Ising model, which was proposed in 1920 by Ernst Ising’s Ph.D. advisor Wilhelm Lenz. This model incorporates only the most basic physics of magnetic materials and operates on the principle that natural systems tend toward minimum energy states. Systems tend toward minimum energy states. The Metropolis algorithm is a common method for numerically constructing equilibrium states of the Ising ferromagnet. In this approach, a randomly-chosen spin either flips or does not flip based on which action lowers or raises the energy $\Delta E$ represents the change in magnetostatic energy from a potential flip (as measured by $H$), and the spin is flipped if $\Delta E < 0$. If $\Delta E > 0$, the probability of the spin flipping is given by the Gibbs factor for $\Delta E$. Sweeping through the whole lattice and iterating the process many times attains a local minimum in the system’s energy. We have adapted the classical Ising model to study and explain the observed geometry of melt pond configurations and capture the fundamental physical mechanism of pattern formation in melt ponds on Arctic sea ice [5]. While previous studies have developed important and instructive numerical models of melt pond evolution [2, 3], we present two examples of such models in Figure 2a. In these models, we formulate the problem of finding the magnetization $M(T, H)$—or order parameter—of an Ising ferromagnet at temperature $T$ in field $H$. The Hamiltonian $H$ with ferromagnetic interaction $J > 0$ between nearest neighbor pairs is given by $H = -J \sum_{i,j} s_i s_j$ for any configuration $\omega$ of $\{-1, 1\}^N$ of the spin variables. The canonical partition function $Z_{\beta}$, which yields the system’s observables, is given by $Z_{\beta}(T, H) = \frac{1}{Z_{\beta}} \exp(-\beta H) = \exp(-\beta N f(T, H))$, where $\beta = 1/kT$, $k$ is Boltzmann’s constant, $\exp(-\beta H)$ is the Gibbs factor, and $f$ is the free energy per site $f(T, H) = (-1/\beta N) \log Z_{\beta}(T, H)$. The magnetization $M(T, H)$ is given by $M(T, H) = \lim_{N \to \infty} \frac{1}{N} \sum_{i,j} s_i s_j$ averaged over $\omega \in \Omega$ with Gibbs’ weights and expressed in terms of the free energy $f(T, H)$ as $M(T, H) = \lim_{N \to \infty} f(T, H)$.
Melt Ponds

Continued from page 5

three models were somewhat detailed and did not focus on the way in which meltwater is distributed over the sea ice surface. Our new model is simplistic and accounts for only the system’s most basic physics. In fact, the only measured parameter is the one-meter lattice spacing, which is determined by snow topography data.

The simulated ponds are metastable equilibria of our melt pond ising model. They have geometrical characteristics that agree very closely with observed scaling of pond sizes [6] and the transition in pond fractal dimension [4]. Researchers have also developed continuum percolation models that reproduce these geometrical features [1, 8].

We aim to use our Ising model to introduce a predictive capability to cyrosphate modeling based on ideas of statistical mechanics and energy minimization, utilizing just the logical processes in global climate models—will deduce such evolutionary rules emerging techniques—like machine learning—will deduce such evolutionary rules from observational data.

References


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