Mathematics 2210 PRACTICE EXAM III Fall 2013

1. Evaluate the integral
$$\iint_R (x+y)dA,$$
where R is the triangle with vertices $(0,0)$, $(0,4)$ and $(1,4)$.

2. Evaluate the iterated integral,
$$\int_0^2 \int_0^{\sqrt{4-x^2}} (x+y)dydx.$$

3. Evaluate the following integral by changing to polar coordinates,
$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} xdxdy.$$

4. Compute the surface area of the bottom part of the paraboloid $z = x^2 + y^2$ that is cut off by the plane $z = 9$.

5. Compute the surface area of the part of the sphere $x^2+y^2+z^2 = a^2$ inside the circular cylinder $x^2 + y^2 = b^2$, where $0 < b \le a$.

6. Compute the volume of the solid in the first octant bounded by $y = 2x^2$ and $y+4z = 8$.

7. Compute the Jacobian $J(r, \theta)$ of the transformation from polar coordinates to Cartesian coordinates given below:
$$x = r\cos\theta$$
$$y = r\sin\theta.$$

8. Compute the Jacobian $J(x, y)$ of the transformation from Cartesian coordinates to polar coordinates given below:
$$r = \sqrt{x^2+y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
Recall: $D_x \tan^{-1} x = \frac{1}{1+x^2}$. What is the relationship between $J(r, \theta)$ and $J(x, y)$?

9. Let $u(x, y) = \log\sqrt{x^2+y^2} = \log r$.
   (a) Find the vector field associated with this scalar field, by computing $\operatorname{grad} u = \nabla u$.
   (b) Compute $\operatorname{curl}(\operatorname{grad} u) = \nabla \times (\nabla u)$.
   (c) What are the level sets?

10. Let $\varphi(x, y) = x^2 - y^2$.
   (a) Compute $\operatorname{grad}\varphi = \nabla\varphi$.

(b) Compute $\operatorname{div}(\operatorname{grad}\varphi) = \nabla \cdot (\nabla\varphi)$.

(c) Based on your findings, what kind of function is φ?

11. Find $\operatorname{div}\mathbf{F}$ and $\operatorname{curl}\mathbf{F}$, where $\mathbf{F}(x,y,z) = x^2\mathbf{i} - 2xy\mathbf{j} + yz^2\mathbf{k}$.

12. Evaluate the following line integral, where C is the curve $x = 3t, y = t^3, 0 \le t \le 1$.

$$\int_C (x^3 + y)ds$$

13. Evaluate the following line integral, where C is the line segment from $(0,0)$ to $(\pi, 2\pi)$.

$$\int_C (\sin x + \cos y)ds$$