Mathematics 1210 PRACTICE EXAM I Fall 2018

1. Calculate the following limits. If a particular limit does not exist, state this clearly and tell why.

   (a) $\lim_{x\to\sqrt{2}} 3x^2$ (b) $\lim_{\theta\to\pi/2} \tan\theta$ (c) $\lim_{x\to -1} \dfrac{x^2-x+2}{x+1}$ (d) $\lim_{x\to 0^+} \sqrt{x}\sin\left(\dfrac{1}{x^2}\right)$

   (e) $\lim_{x\to 0} \dfrac{\sin(x^2)}{x}$ (f) $\lim_{x\to +\infty} \dfrac{\sin x}{x}$ (g) $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^3, & x \le 2 \\ x, & x > 2 \end{cases}$

   (h) $\lim_{x\to\pi} f(x)$, where $f(x) = \begin{cases} 0, & x \text{ irrational} \\ \sin\left(\frac{1}{q}\right), & x = \frac{p}{q} \text{ rational} \end{cases}$ (i) $\lim_{x\to +\infty} \sqrt[3]{\dfrac{8x^7+3x^5}{x^7+6x^2}}$

   (j) $\lim_{x\to 0} \dfrac{\sin x}{x^3}$

2. (a) Let $f(x) = \sqrt{x}$. Using the *definition* of the derivative, calculate $f'(x)$. Do the same for $g(x) = x^2$.

   (b) Using your result from (a), find the equation of the line tangent to the graph of $f(x) = \sqrt{x}$ at $x = 1$. Do the same for $g(x) = x^2$.

3. Let $f(x) = -x$ when $x \le 0, x \ne -1$; 2 when $x = -1$; $\sqrt{x}$ when $0 < x < 1$; $\sqrt[3]{3-x}$ when $x \ge 1$. Sketch the graph of $f(x)$.

   (a) For which points c does $\lim_{x\to c} f(x)$ exist? (b) For which points is f continuous?

   (c) For which points is f differentiable?

4. Let $f(x) = x+2$ when $x \le 0$; $-\dfrac{1}{2}x + 2$ when $0 < x \le 2$; $\sqrt{x-2} + 1$ when $x > 2$. Sketch the graph of $f(x)$, and then using your result sketch the graph of $f'(x)$.

5. Find the derivative and of

   (a) $f(x) = 12x^5 + 5x^4 + x^2 + 2x + 1$

   (b) $f(x) = \tan x$

   (c) $f(x) = (3x^2 - 2x + 1)(x - 1)$

   (d) $f(x) = x\cos x$

   (e) $f(x) = \dfrac{x^2+1}{x+\pi}$

6. Let the position $x(t)$ of a particle at time t be given by $x(t) = 3t^2 - 2t + 1$. Find the instantaneous velocity $v(t)$ of the particle for any time t. Where is the particle when its velocity is zero?

7. (a) Find the equation of the line containing the two points $P = (-1, 2)$ and $Q = (1, 1)$ in the form $y = mx + b$.

   (b) Find the derivative $\dfrac{dy}{dx}$ of the expression for $y(x)$ you found in (a).