Practice Final

Math 1010-001
August 1, 2005

Name: 

This is a practice exam. The final exam will resemble this exam but, will not be exactly like this. Memorizing solutions to these problems will not help you. It is intended to give you a general idea of what I might ask on the final exam.

Read all of the following information before starting the exam:

• Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
• This exam is closed book and no calculators are allowed.
• Justify your answers algebraically whenever possible to ensure full credit. You may also use pictures whenever appropriate. Remember that you are trying to communicate to me that you understand the material. I am not a mind reader.
• Circle or otherwise indicate your final answers.
• Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
• This test has 28 problems and is worth 226 points,
• Good luck!
1. (10 points) Solve the following. Your answer should be a fraction like $x = a/b$ where $a$ and $b$ are integers.

$$\frac{x - \frac{1}{3}}{\frac{1}{12} + \frac{3}{12}} = x$$

$$\frac{x - \frac{1}{3}}{\frac{1}{12}} = \frac{x - \frac{1}{3}}{\frac{1}{3}} = x = \left(x - \frac{1}{3}\right)3 = x$$

$$= 7 \quad 3x - 1 = x = 7 \quad 2x = 1 \quad \boxed{x = \frac{1}{2}}$$

*What would happen if there was an $x^2$ somewhere?*

2. (8 points) Find the values of $x$ for which the following inequality is true.

$$x + 7 > 3x + 2$$

$$-x - 7 = -x - 7$$

$$5 + 0 > 2x - 5 + 5$$

$$\frac{5}{2} \quad \frac{2x}{2} = \frac{5}{2} \quad \boxed{x > \frac{5}{2}}$$
3. (12 points) Solve the following story problems. Be sure to make it clear how you solved the problem. Write down exactly what you have done. Remember, you are trying to communicate to me that you understand how to do this.

a. (6 pts) You and a coworker together make $38.00 per hour. Your coworker earns 10% less than you do. What is your hourly wage?

\[ m = \text{me} \]
\[ c = \text{coworker} \]
\[ m + c = 38 \quad \text{line forms} \]
\[ c = m - 0.1m = 0.9m \]
10% less than m

Subs
\[ m + 0.9m = 38 \]
\[ 1.9m = 38 \]
\[ m = \frac{38}{1.9} = 20 \]
You coworker makes $15.00 per hour (but the question does not ask for it)

b. (6 pts) After taking Math 1010 your productivity goes up. This increase in productivity has earned your company more money. So your boss has decided to give herself a raise. She used to make $40.00 per hour. Now she makes $65.00 per hour. This is a raise of what percent?

\[ 40.00 + x \times 40.00 = 65.00 \]
\[ x \times 40.00 = 25.00 \]
\[ x = \frac{25}{40} = \frac{3}{8} = 0.3125 \quad \text{62.5%} \]
4. (12 points)
   a. (6 pts) What is the distance between (2, 3) and (4, -1)?
      \[ d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(2 - 4)^2 + (3 - (-1))^2} = \sqrt{4 + 16} = 4.4 \]
   b. (6 pts) What is the slope of the line that passes through (2, 3) and (4, -1)?
      \[ m = \frac{\sqrt{20}}{2} = \frac{\sqrt{5} \cdot 2}{2} = \sqrt{5} \]

5. (12 points) Graph the following lines.
   a. (4 pts) \[ f(x) = -2x + 0 \]
   b. (4 pts) \[ f(x) = -2x^2 + 1 \]
   c. (4 pts) \[ f(x) = -2x + (x - 3) \]
6. (8 points) \( y = \frac{-x}{2} - 3 \) is a line with a \( y \)-intercept at \((0, -\frac{3}{2})\) and slope \(-\frac{1}{2}\).

7. (8 points)
   a. (4 pts) Find the equation of a line perpendicular to \( y = \frac{-x}{2} - 3 \) that passes through the point \((1, 3)\).

   \[
   \begin{align*}
   \text{Since it is } & \perp \quad m = 2 \\
   y & = 2x + b \quad \Rightarrow \quad 3 = 2 + b \quad \Rightarrow \quad b = 1
   \end{align*}
   \]

   \[
   Y = 2x + 1
   \]

   b. (4 pts) Find the equation of a line parallel to \( y = \frac{-x}{2} - 3 \) that passes through the point \((3, 1)\).

   \[
   \begin{align*}
   y & = \frac{-x}{2} + b \\
   \uparrow & \quad \parallel \text{parallel} \\
   1 & = \frac{-3}{2} + b \quad \sqrt{b = \frac{5}{2} = 2.5}
   \end{align*}
   \]

   \[
   Y = -\frac{x}{2} + \frac{5}{2}
   \]
8. (8 points) What is the domain and range of the following function?

\[ f(x) = \sqrt{-4x + 8 + 2} \]

\[-4x + 8 \geq 0 \quad \text{(Since it is inside a square root)}
\]
\[ 6 \geq 4x \]
\[ x \leq \frac{3}{2} \]
\[ \text{Domain} \]

The cut off of \[ \sqrt{-4x+8} \geq 0 \]
using graph

\[ \sqrt{-4x+8} + 2 \geq 2 \]
\[ f(x) \geq 2 \]
\[ \text{Range} \]

9. (12 points) Let \( f(x) = x^2 - 2x + 1 \). Answer the following questions in the simplest possible form.

a. (4 pts) What is \( f(10) \)?

\[ f(10) = 100 - 20 + 1 = 81 \]

b. (4 pts) What is \( f(-1) \)?

\[ f(-1) = (-1)^2 - 2(-1) + 1 = 1 + 2 + 1 = 4 \]

c. (4 pts) What is \( f(x^2 - 1) \)?

\[ f(x^2 - 1) = (x^2 - 1)^2 - 2(x^2 - 1) + 1 \]
\[ \quad = x^4 - 2x^2 + 1 - 2x^2 + 2 - 1 \]
\[ \quad = x^4 - 4x^2 + 4 \]
10. (10 points) Your friend goes to the auto parts store to get a distributor cap, a rotor and an ignition coil. She tells you that together they were $60 and the rotor is $5 less than half the price of the cap. The coil is twice as much as the distributor cap. Instead of asking your friend how much each item was, you decide to figure it out yourself, so you look cool.

a. (5 pts) Set up a system of linear equations that describe this situation. (don't solve them)

\[
\begin{align*}
\frac{d}{2} - 5 &= r \\
2d &= c \\
d + r + c &= 60
\end{align*}
\]

b. (5 pts) Solve the system of linear equations using your method of choice.

Substituting

\[
d + \frac{d}{2} - 5 + 2d = 66
\]

\[
3d + \frac{d}{2} = 65 \\
3.5d = 65
\]

\[
d = \frac{65}{3.5} = 18.57 \\
c = 2 \cdot 18.57 = 37.14
\]

\[
r = \frac{18.57}{2} - 5 = 4.29
\]
11. (14 points) Rewrite the following expressions using just one rational exponent or if possible write it as an integer. (Be sure to study these. Many of you missed easy points for not knowing how to do these)
   a. (4 pts) \(64^{\frac{3}{4}} = (8^{3})^{\frac{2}{4}} = (2^3)^{\frac{2}{3}} = (2^2)^2 = 4^2 = 16\)
   b. (4 pts) \((3^2)^{\frac{9}{4}} = \frac{27}{4} = \sqrt[4]{\frac{3^2}{4}}\)
   c. (3 pts) \((2^{\frac{3}{5}})^{\frac{7}{3}}\)
   d. (3 pts) \(\frac{u^4 v^3}{u^3} = u^{\frac{4}{3} - \frac{3}{4}} = u^{\frac{5}{3} - \frac{3}{4}} = u^{\frac{20}{12} - \frac{9}{12}} = u^{\frac{11}{12}}\)

12. (4 points) Write 2,030,400,000,000 in scientific notation.
\[2.03 \times 10^{12}\]

13. (8 points) Let the polynomial \(p\) be defined by
\[p(x) = (9x - 4)(2x + 3)\]
a. (4 pts) Write \(p\) in standard form \((p(x) = ax^2 + bx + c)\).
\[18x^2 - 8x + 27x - 12\]
\[18x^2 + 19x - 12\]
b. (4 pts) What is \(p(4/9)\)?
\[p\left(\frac{4}{9}\right) = \left(9 \cdot \frac{4}{9} - 4\right)\left(2 \cdot \frac{4}{9} + 7\right)\]
\[= (4 - 4)\left(2 \cdot \frac{4}{9} + 7\right)\]
\[= 0\]
14. (10 points) On video games it is not uncommon for characters to jump their own height and more. For a 6 foot tall human how fast would they have to leave the ground to jump 9 feet (150% of their height) into the air. Remember that
\[ h(t) = h_0 + v_0 t - 16t^2 \]
\[ v(t) = v_0 - 32t \]

\[ h(t) = 0 + v_0 t - 16t^2 \]
\[ v(t) = v_0 - 32t \]

\[ v(t) = 0 \text{ (highest point)} \]
\[ h(t) = 9 \text{ (highest point)} \]

\[ 0 = v_0 - 32t = 7 \quad 32t = 16 \quad t = \frac{v_0}{32} \]

\[ 9 = v_0 \cdot \frac{v_0}{32} - 16 \frac{v_0^2}{32^2} = \frac{v_0^2}{32} - \frac{v_0^2}{64} \]

\[ = \frac{V_0^2}{64} \quad \Rightarrow \quad 9 \cdot \frac{64}{64} = V_0^2 \]

\[ = \quad v_0 = \sqrt{9 \cdot \frac{64}{64}} = \sqrt{9} \cdot \sqrt{\frac{64}{64}} = \sqrt{9} = 3 \cdot 8 = 24 \]

(Be sure to study the homework problem 28-31 from assignment 6).
15. (10 points) Simplify
\[ \frac{1}{x - 3} - \frac{7}{x^2 + x - 30} \]
\[ \text{LCM} = (x - 3)(x + 6)(x - 5) \]
\[ \frac{1}{x - 3} \cdot \frac{(x + 6)(x - 5)}{(x + 6)(x - 5)} - \frac{7}{(x + 6)(x - 5)} \cdot \frac{(x - 3)}{(x - 3)} \]
\[ = \frac{(x + 6)(x - 5) - 7(x - 3)}{(x + 6)(x - 5)(x - 3)} \]
\[ = \frac{x^2 + x - 30 - 7x + 21}{(x + 6)(x - 5)(x - 3)} \]
\[ = \frac{x^2 - 6x - 9}{(x + 6)(x - 5)(x - 3)} \]
\[ = \frac{x^2 - 6x - 9}{(x^2 + x - 30)} \]

16. (10 points) Simplify
\[ \frac{1}{x^2 + x - 30} \cdot \frac{1}{x + 4} \]
\[ = \frac{(x + 4)}{(x + 6)(x - 5)} \]
17. (10 points) A circular swimming pool has a depth \( d \) and a radius that is 10 times its depth. A square pool has the same depth as the circular pool its width 2 less than 10 times its depth. What is the ratio of the square pool’s volume to the circular pools volume? (This problem was missed by too many of you. Something like it will be on the test. )

\[
V_c = \pi r^2 d = \pi (10d)^2 d \\
V_s = \text{ewd} = (10d-2)^2 d \\
\frac{V_c}{V_s} = \frac{(10d-2)^2 d}{\pi (10d)^2 d} = \frac{(10d-2)^2}{100\pi \cdot d^2}
\]

18. (10 points) Solve the following system of equations: (Many of you did not even know how to start this one. We spent more than a week on this type of problem and we developed three strategies to deal with it. Matrix elimination is the best of these. You could do elimination with the equations. Substitution is usually a bad method to use on problems like this one.)

\[
\begin{align*}
x & - y + z = 1 \\
-x & + y - 2z = 2 \\
-x & - y = 1
\end{align*}
\]

\[
\begin{bmatrix}
1 & 0 & 1 & 1 \\
-1 & 1 & -2 & 2 \\
-1 & -1 & 0 & 1
\end{bmatrix}
\]

\[
R_1 + R_2 \\
R_1 + R_3
\]

\[
= \begin{bmatrix}
1 & 0 & 1 & 1 \\
0 & 1 & -1 & 3 \\
0 & 1 & 1 & 2
\end{bmatrix}
\]

\[
R_1 + R_3
\]

\[
= \begin{bmatrix}
1 & 0 & 1 & 1 \\
0 & 1 & 1 & 3 \\
0 & 0 & 0 & 5
\end{bmatrix}
\]

The solution
19. (10 points) You have just landed on a small planet where gravity is \( \frac{1}{16} \) what it is on earth. Because of this the projectile motion equations become:

\[
h(t) = h_0 + v_0 t - \frac{1}{2} t^2
\]

\[
v(t) = v_0 - 2t
\]

To celebrate your landing you climb to the top of your 30 foot tall spacecraft throw a rock straight up in the air with an initial velocity of 70 ft/sec. When does the rock hit the ground? (\( h_0 \) is the height when the rock is thrown. \( v_0 \) is the velocity when the rock is thrown. \( h(t) \) is the height at the time \( t \). \( v(t) \) is the velocity at time \( t \). Notice that this problem is different than the one on the practice exam and on Exam 2.)

\[
h_0 = 30 \quad v_0 = 70
\]

\[
h(t) = 30 + 70 t - \frac{1}{2} t^2
\]

\[
v(t) = 70 - 2t
\]

\[
0 = 30 + 70 t - \frac{1}{2} t^2
\]

Using the quadratic formula:

\[
-70 \pm \sqrt{70^2 - 4 \cdot \frac{1}{2} \cdot 30}
\]

\[
= -70 \pm \sqrt{5020}
\]

\[
= 70 \pm \sqrt{4.1255}
\]

These are two answers but we were asked for one answer. So pick the positive one since that is the future (after you throw the rock).

\[
t = 70 + \frac{\sqrt{1255}}{2}
\]

Rock hits the ground

\[
\text{meaning } h(t) = 0
\]
20. (4 points) Solve the following equation.

\[ \sqrt{3x - 4} = (7 + x)^2 \]

\[ 3x - 4 = 49 + 14x + x^2 \]

\[ 0 = x^2 + 11x + 53 \]

\[ x = \frac{-11 \pm \sqrt{121 - 4 \cdot 53}}{2} \]

\[ x = \frac{-11 \pm \sqrt{-92}}{2} \]

These are complex!

These cannot be solution to a real equation

No solution.

21. (4 points) Let \( p(x) = x^2 + x - 56 \). Find the roots of this polynomial.

Factor: \( (x - 8)(x + 7) \)

\[ x = 8, 7 \]
22. (4 points) let \( p(x) = 2x^2 - x + 1 \). Find the roots of this polynomial.

23. (2 points) When can you divide by zero?

\[ \text{Never} \]

24. (8 points)
   a. (2 pts) Multiply \((3 + i) \times (1 + 6i)\).

\[
\begin{align*}
3 + i + 18i + 6i^2 &= 3 + 19i - 6 \\
&= -3 + 19i
\end{align*}
\]

b. (2 pts) Divide \((2 + i) ÷ (2 - i)\)

\[
\frac{2 + i}{2 - i} = \frac{4 + 4i + i^2}{4 + 1} = \frac{4 + 4i - 1}{5} = \frac{3 + 4i}{5}
\]

c. (2 pts) Add \((-2 - 6i) + (7 + 6i)\)

\[
-2 + 7 - 6i + 6i = 5 + 0i = 5
\]

d. (2 pts) Subtract \((-2 - 6i) - (7 + 6i)\)

\[
\begin{array}{c}
-2 - 7 - 6i - 6i \\
\hline
-9 - 12i
\end{array}
\]

25. (2 points) What is a function?

A function is an object that takes an input and returns a unique output.
26. (2 points) What is the Golden Rule of Algebra?

27. (2 points) If I have three variables how many equations do I need to find values for all three variables?

three

28. (12 points) Divide $x^4 - x + 2$ by $x + 2$ using any method you like.

\[
\begin{array}{c|cccc}
\multicolumn{2}{c|}{-2} & 1 & 0 & -1 \24 & -8 & 16 \\
\hline
1 & -24 & -9 & 20 \\
\end{array}
\]

\[
x^3 - 2x^2 + 4x - 9 + \frac{20}{x+2}
\]
**Bonus Question** You are given a magic penny that doubles every day. Also, any penny that was made by the magic penny is also a magic penny. Let's say Lucky the Leperachaun gave one to you on August 1. How much money do you have on August 31? (You do not have to write your answer in simplest form. Do not try to find a decimal representation for this number.)

\[
p(t) = 1 \times 2^{t-1}
\]

\[
p(31) = 1 \times 2^{30} = 1 \times 2^{30} = \frac{1}{2} \text{ million}
\]