Assignment 8 Solutions

Section 9.4 : 1, 4, 5, 7, 11, 14, 17, 33

1) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+2n+3}$ $\qquad \displaystyle\frac{n}{n^2+2n+3} \sim \frac{n}{n^2} = \frac{1}{n} = b_n$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{n}{n^2+2n+3}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{n^2+2n+3} = \lim_{n\to\infty} \frac{1}{1+\frac{2}{n}+\frac{3}{n^2}} = 1 = L$$

$\Rightarrow 0 < L < \infty \Rightarrow a_n$ and b_n behave together

$b_n = \frac{1}{n}$ = harmonic series diverges $\Rightarrow \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2n+3}$ $\boxed{\text{diverges}}$

4) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{2n+1}}{n^2}$ $\qquad \displaystyle\frac{\sqrt{2n+1}}{n^2} \sim \frac{2\sqrt{n}}{n^2} = \frac{2}{n^{3/2}} = b_n$

$$\Rightarrow L = \lim_{n\to\infty} \frac{\frac{\sqrt{2n+1}}{n^2}}{\frac{2}{n^{3/2}}} = \lim_{n\to\infty} \frac{n^{3/2}\sqrt{2n+1}}{2n^2}$$

$$= \lim_{n\to\infty} \frac{\sqrt{2n+1}}{2\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{2+\frac{1}{n}}}{2\sqrt{1}} = \frac{\sqrt{2}}{2} = L$$

$\Rightarrow 0 < L < \infty$, b_n = p-series $p = 3/2 > 1 \Rightarrow$ converges

$$\Rightarrow \sum_{n=1}^{\infty} \frac{\sqrt{2n+1}}{n^2} \quad \boxed{\text{converges}}$$

5) $\sum_{n=1}^{\infty} \frac{8^n}{n!} \Rightarrow a_n = \frac{8^n}{n!}, \quad a_{n+1} = \frac{8^{n+1}}{(n+1)!}$

$$R = \lim_{n\to\infty} \frac{\frac{8^{n+1}}{(n+1)!}}{\frac{8^n}{n!}} = \lim_{n\to\infty} \frac{8^{n+1}\, n!}{8^n (n+1)!} = \lim_{n\to\infty} \frac{8}{n+1} = \frac{8}{\infty} = 0$$

$R < 1 \Rightarrow \sum_{n=1}^{\infty} \frac{8^n}{n!}$ $\boxed{\text{Converges}}$

7) $\sum_{n=1}^{\infty} \frac{n!}{n^{100}} \Rightarrow a_n = \frac{n!}{n^{100}}, \quad a_{n+1} = \frac{(n+1)!}{(n+1)^{100}}$

$$R = \lim_{n\to\infty} \frac{\frac{(n+1)!}{(n+1)^{100}}}{\frac{n!}{n^{100}}} = \lim_{n\to\infty} \frac{n^{100}(n+1)!}{(n+1)^{100}\, n!} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{100} (n+1)$$

$$= 1 \cdot (\infty + 1) = \infty$$

$\Rightarrow \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty \Rightarrow \sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ diverges

11) $\sum_{n=1}^{\infty} \frac{n}{n+200} \Rightarrow \lim_{n\to\infty} \frac{n}{n+200} = \lim_{n\to\infty} \frac{1}{1+\frac{200}{n}} = 1 \neq 0$

$\Rightarrow$ diverges by the n^{th} term test

14) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+1}$ $\qquad \frac{\sqrt{n+1}}{n^2+1} \sim \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}} = b_n$

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n+1}}{n^2+1}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{n^{3/2}\sqrt{n+1}}{n^2+1} = \lim_{n\to\infty} \frac{n^{3/2}\sqrt{n+1}}{n^2+1} \cdot \frac{\frac{1}{n^{3/2}}}{\frac{1}{n^{3/2}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n} + \frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n} + n^{-3/2}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{1+\frac{1}{n}}}{1+\frac{1}{n^2}} = \frac{\sqrt{1}}{1} = 1 = L$$

$0 < L < \infty$, $b_n = \frac{1}{n^{3/2}}$ = p-series $p = 3/2 > 1 \Rightarrow$ converges

$\Rightarrow \sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+1}$ $\boxed{\text{converges}}$ by the Limit Comparison Test

17) $\sum_{n=1}^{\infty} \frac{4n^3+3n}{n^5-4n^2+1}$ $\qquad \frac{4n^3+3n}{n^5-4n^2+1} \sim \frac{4n^3}{n^5} = \frac{4}{n^2} = b_n$

$$L = \lim_{n\to\infty} \frac{\frac{4n^3+3n}{n^5-4n^2+1}}{\frac{4}{n^2}} = \lim_{n\to\infty} \frac{(4n^3+3n)n^2}{4(n^5-4n^2+1)}$$

$$= \lim_{n\to\infty} \frac{4n^5+3n^3}{4n^5-16n^2+4} = \lim_{n\to\infty} \frac{4+\frac{3}{n^2}}{4-\frac{16}{n^3}+\frac{4}{n^5}} = 1 = L$$

$0 < L < \infty$, $b_n = \frac{4}{n^2}$ = p-series, $p = 2 > 1 \Rightarrow$ converges

$\Rightarrow \sum_{n=1}^{\infty} \frac{4n^3+3n}{n^5-4n+1}$ converges by Limit Comparison Test

33) $\sum_{n=1}^{\infty} \frac{4^n + n}{n!}$ there is a factorial so try ratio test

$$a_n = \frac{4^n + n}{n!} \quad , \quad a_{n+1} = \frac{4^{n+1} + n+1}{(n+1)!}$$

$$R = \lim_{n\to\infty} \frac{\frac{4^{n+1} + (n+1)}{(n+1)!}}{\frac{4^n + n}{n!}} = \lim_{n\to\infty} \frac{n!\,(4^{n+1} + n + 1)}{(n+1)!\,(4^n + n)}$$

$$= \lim_{n\to\infty} \frac{4^{n+1} + n + 1}{(n+1)(4^n + n)}$$

leading order is 4^n

exponential > polynomial

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{4^n}} \frac{(4^{n+1} + n + 1)}{(n+1)(4^n + n)} = \lim_{n\to\infty} \frac{4 + \frac{n}{4^n} + \frac{1}{4^n}}{(n+1)\left(1 + \frac{n}{4^n}\right)}$$

$$= \lim_{n\to\infty} \frac{4 + \cancel{\frac{n}{4^n}}^{\to 0} + \cancel{\frac{1}{4^n}}^{\to 0}}{\cancel{\frac{n^2}{4^n}}^{\to 0} + n + 1 + \cancel{\frac{n}{4^n}}^{\to 0}}$$

note $\frac{n}{4^n} \to 0$

$\frac{n^2}{4^n} \to 0$

$\frac{1}{4^n} \to 0$

$$= \frac{4}{\infty + 1} = \frac{4}{\infty} = 0 = R$$

$R < 1 \Rightarrow$ series $\boxed{\text{converges}}$ by ratio test

Section 9.5: 1, 4, 5, 7, 10, 11, 13, 18, 23

1) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{3n+1}$ → alternating

$\lim_{n\to\infty} \frac{2}{3n+1} = \frac{2}{\infty} = 0$

Sequence → 0

$a_{n+1} = \frac{2}{3(n+1)+1} = \frac{2}{3n+4}$ $\quad a_n = \frac{2}{3n+1}$

$\Rightarrow \quad a_n > a_{n+1} > 0 \quad \Rightarrow$ series $\boxed{\text{converges}}$

$\text{error} \approx |a_{10}| = \frac{2}{3(10)+1} = \boxed{\frac{2}{31}}$

4) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$ $\quad a_n = \frac{n}{n^2+1}$ $\quad a_{n+1} = \frac{n+1}{(n+1)^2+1}$

$\frac{n}{n^2+1} > \frac{n+1}{n^2+2n+2} \Rightarrow a_n > a_{n+1} > 0$

$\lim_{n\to\infty} \frac{n}{n^2+1} = \frac{\frac{1}{n}}{1+\frac{1}{n^2}} = \frac{0}{1} = 0 \Rightarrow$ Sequence → 0

$\Rightarrow$ alternating series $\boxed{\text{converges}}$

$\text{error} \approx |a_{10}| = \frac{10}{10^2+1} = \boxed{\frac{10}{101}}$

5) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$ $\quad |a_n| = \frac{\ln(n)}{n} = f(n)$

$f(x) = \frac{\ln(x)}{x}$, $\lim_{x\to\infty} \frac{\ln(x)}{x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{\ln(x)}{x} \overset{L}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = 0$

$\Rightarrow$ alternating series converges

error $\approx$ $|a_{10}| = \left| \frac{(-1)^{11} \ln(10)}{10} \right| = \boxed{\frac{\ln(10)}{10}}$

7) $\sum_{n=1}^{\infty} \left(\frac{-3}{4}\right)^n = \sum_{n=1}^{\infty} (-1)^n \left(\frac{3}{4}\right)^n$

$\sum_{n=1}^{\infty} \left| \left(\frac{-3}{4}\right)^n \right| = \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n = \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \cdots$

$=$ geometric series, $a = \frac{3}{4}$ $r = \frac{3}{4}$

$\left|\frac{3}{4}\right| < 1 \Rightarrow$ converges

since $\sum_{n=1}^{\infty} \left| \left(\frac{-3}{4}\right)^n \right|$ converges, $\sum_{n=1}^{\infty} \left(\frac{-3}{4}\right)^n$ converges absolutely

10) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{e^n}$ $\quad$ Absolute Ratio Test

$|a_n| = \frac{n^2}{e^n}$, $|a_{n+1}| = \frac{(n+1)^2}{e^{n+1}}$

$R = \lim_{n\to\infty} \frac{\frac{(n+1)^2}{e^{n+1}}}{\frac{n^2}{e^n}} = \lim_{n\to\infty} \frac{(n+1)^2 e^n}{n^2 e^{n+1}} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^2 \frac{1}{e}$

$= \lim_{n\to\infty} \left(\frac{1 + \cancel{\frac{1}{n}}^{\,0}}{1}\right)^2 \frac{1}{e} = \frac{1}{e} < 1$ $\Rightarrow R < 1$ converge absolutely

11) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n(n+1)}$

Absolute Ratio Test

$|a_n| = \frac{1}{n(n+1)} \qquad |a_{n+1}| = \frac{1}{(n+1)(n+2)}$

$$R = \lim_{n\to\infty} \frac{\frac{1}{(n+1)(n+2)}}{\frac{1}{n(n+1)}} = \lim_{n\to\infty} \frac{n\cancel{(n+1)}}{\cancel{(n+1)}(n+2)} = \lim_{n\to\infty} \frac{n}{n+2} = 1$$

$\Rightarrow$ test inconclusive

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n(n+1)} \right| = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n^2+n}$$

$\frac{1}{n^2+n} \approx \frac{1}{n^2} = b_n \Rightarrow$ limit comparison test

$$L = \lim_{n\to\infty} \frac{\frac{1}{n^2+n}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2+n} = \lim_{n\to\infty} \frac{1}{1+\cancel{\frac{1}{n}}^{\,\to 0}} = 1$$

$0 < L < \infty \Rightarrow a_n, b_n$ converge / diverge together

$b_n = \frac{1}{n^2} =$ p-series, $p = 2 \Rightarrow$ converges

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2+n}$ converges

$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)}$ converges absolutely

13) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{5n}$, $\lim_{n\to\infty} \frac{1}{5n} = 0$ and $\frac{1}{5n} > \frac{1}{5(n+1)} > 0$

$\Rightarrow$ converges

$$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{5n} \right| = \sum_{n=1}^{\infty} \frac{1}{5n} = \frac{1}{5} \sum_{n=1}^{\infty} \frac{1}{n}$$ = harmonic series diverges

$\Rightarrow$ $\boxed{\text{conditionally convergent}}$

18) $$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n(1+\sqrt{n})}$$

$$\lim_{n\to\infty} \frac{1}{n(1+\sqrt{n})} = \frac{1}{\infty(1+\infty)} = \frac{1}{\infty} = 0$$

$$\frac{1}{n(1+\sqrt{n})} > \frac{1}{(n+1)(1+\sqrt{n+1})} > 0 \Rightarrow \text{convergent}$$

$$\sum_{n=1}^{\infty} \left| (-1)^n \frac{1}{n(1+\sqrt{n})} \right| = \sum_{n=1}^{\infty} \frac{1}{n(1+\sqrt{n})} = \sum_{n=1}^{\infty} \frac{1}{n+n\sqrt{n}}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n+n^{3/2}}, \quad \frac{1}{n^{3/2}+n} \approx \frac{1}{n^{3/2}}$$

Limit comparison test

$$L = \lim_{n\to\infty} \frac{\frac{1}{n^{3/2}+n}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{n^{3/2}}{n^{3/2}+n} = \lim_{n\to\infty} \frac{1}{1+\cancel{\frac{1}{\sqrt{n}}}_{\to 0}} = 1$$

$0 < L < \infty \Rightarrow a_n, b_n$ converge / diverge together

$b_n = \frac{1}{n^{3/2}}$ = p-series, $p = 3/2 > 1 \Rightarrow$ converge

$$\Rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{1}{n(1+\sqrt{n})} \; \boxed{\text{converges absolutely}}$$

23) $$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \text{alternating harmonic}$$

$$\boxed{\text{Converges conditionally}}$$

Section 9.6: 1, 3, 4, 9, 11, 12, 17, 25

1) $\sum_{n=1}^{\infty} \frac{x^n}{(n-1)!}$ Absolute Ratio Test

$$|a_n| = \left|\frac{x^n}{(n-1)!}\right| = \frac{|x|^n}{(n-1)!}$$

$$|a_{n+1}| = \left|\frac{x^{n+1}}{n!}\right| = \frac{|x|^{n+1}}{n!}$$

$$R = \lim_{n\to\infty} \frac{\frac{|x|^{n+1}}{n!}}{\frac{|x|^n}{(n-1)!}} = \lim_{n\to\infty} \frac{(n-1)!\,|x|^{n+1}}{n!\,|x|^n} = \lim_{n\to\infty} \frac{|x|}{n} = \frac{|x|}{\infty} = 0$$

$R = 0 \;\; \forall x \;\; \Rightarrow \;\; R < 1 \Rightarrow$ converge absolutely

$$\boxed{\text{Convergence Set} = \mathbb{R}}$$

3) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ A.R.T $\Rightarrow$ $|a_n| = \left|\frac{x^n}{n^2}\right| = \frac{|x|^n}{n^2}$

$$|a_{n+1}| = \left|\frac{x^{n+1}}{(n+1)^2}\right| = \frac{|x|^{n+1}}{(n+1)^2}$$

$$R = \lim_{n\to\infty} \frac{\frac{|x|^{n+1}}{(n+1)^2}}{\frac{|x|^n}{n^2}} = \lim_{n\to\infty} \frac{n^2|x|^{n+1}}{(n+1)^2|x|^n} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^2 |x|$$

$= |x| = R < 1$ for convergence

Check $|x| = R = 1 \Rightarrow x = \pm 1$

$x = 1$, $\sum_{n=1}^{\infty} \frac{1^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} =$ p-series, $p = 2$
Converges

when $x=-1$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ = alternating p-series, $p=2$ converges

$\Rightarrow$ Convergence Set $\boxed{-1 \le x \le 1}$

4) $\sum_{n=1}^{\infty} nx^n$ $\qquad |a_n| = n|x|^n$, $|a_{n+1}| = (n+1)|x|^{n+1}$

$$R = \lim_{n\to\infty} \frac{(n+1)|x|^{n+1}}{n|x|^n} = \lim_{n\to\infty} \underbrace{\left(\frac{n+1}{n}\right)}_{=1}|x| = |x| = R$$

$R = |x| < 1$ for convergence

Check $R = |x| = 1$, $\quad x=1$ $\sum_{n=1}^{\infty} n1^n = \sum_{n=1}^{\infty} n$ diverges

$x=-1$ $\sum_{n=1}^{\infty} n(-1)^n$ diverges

$\Rightarrow$ $\boxed{-1 < x < 1 = \text{convergence set}}$

9) $\frac{x}{1\cdot 2} - \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} - \cdots$

$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n(n+1)}$ $\qquad |a_n| = \frac{|x|^n}{n(n+1)}$, $|a_{n+1}| = \frac{|x|^{n+1}}{(n+1)(n+2)}$

$$R = \lim_{n\to\infty} \frac{\frac{|x|^{n+1}}{(n+1)(n+2)}}{\frac{|x|^n}{n(n+1)}} = \lim_{n\to\infty} \frac{n\cancel{(n+1)}|x|^{n+1}}{\cancel{(n+1)}(n+2)|x|^n} = \lim_{n\to\infty} \underbrace{\left(\frac{n}{n+2}\right)}_{1}|x|$$

$R = |x| < 1$ for convergence

Check $R = |x| = 1 \Rightarrow x = \pm 1$

$x = 1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(1)^n}{n^2+n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+n}$$

$$\lim_{n\to\infty} \frac{1}{n^2+n} = 0 \Rightarrow \text{converges}$$

$x = -1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^n}{n^2+n} = \sum_{n=1}^{\infty} \frac{1}{n^2+n} \leq \sum_{n=1}^{\infty} \frac{1}{n^2}$$

which converges (p - series $p = 2$)

$\Rightarrow$ converges by ordinary comparison test

$\Rightarrow$ Convergence Set $\boxed{-1 \leq x \leq 1}$

11) $$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} = \sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!}(-1)^{n+1}$$

$$|a_n| = \frac{|x|^{2n-1}}{(2n-1)!}, \quad |a_{n+1}| = \frac{|x|^{2n+1}}{(2n+1)!}$$

$$R = \lim_{n\to\infty} \frac{\frac{|x|^{2n+1}}{(2n+1)!}}{\frac{|x|^{2n-1}}{(2n-1)!}} = \lim_{n\to\infty} \frac{(2n-1)!\,|x|^{2n+1}}{(2n+1)!\,|x|^{2n-1}} = \lim_{n\to\infty} \frac{|x|^2}{(2n+1)(2n)}$$

$= 0 = R < 1$ for all x

$\Rightarrow$ convergence set $= \boxed{\mathbb{R}}$

12) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$|a_n| = \frac{|x|^{2n}}{(2n)!}$ $\quad |a_{n+1}| = \frac{|x|^{2n+2}}{(2n+2)!}$

$$R = \lim_{n\to\infty} \frac{\frac{|x|^{2n+2}}{(2n+2)!}}{\frac{|x|^{2n}}{(2n)!}} = \lim_{n\to\infty} \frac{|x|^{2n+2}(2n)!}{|x|^{2n}(2n+2)!} = \lim_{n\to\infty} \frac{|x|^2}{(2n+2)(2n+1)}$$

$$= 0 \quad \text{for all } x$$

$R = 0 < 1 \Rightarrow$ convergence set $= \boxed{\mathbb{R}}$

17) $1 - \frac{x}{1\cdot 3} + \frac{x^2}{2\cdot 4} - \frac{x^3}{3\cdot 5} + \cdots$

$= 1 + \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n(n+2)}$, $\quad |a_n| = \frac{|x|^n}{n(n+2)}$

$|a_{n+1}| = \frac{|x|^{n+1}}{(n+1)(n+3)}$

$$R = \lim_{n\to\infty} \frac{\frac{|x|^{n+1}}{(n+1)(n+3)}}{\frac{|x|^n}{n(n+2)}} = \lim_{n\to\infty} \frac{|x|^{n+1} n(n+2)}{|x|^n (n+1)(n+3)} = \lim_{n\to\infty} \frac{|x|(n^2+2n)}{(n^2+4n+3)}$$

$$= \lim_{n\to\infty} |x| \left(\frac{1 + \cancel{\frac{2}{n}}^{\to 0}}{1 + \cancel{\frac{4}{n}}^{\to 0} + \cancel{\frac{3}{n^2}}^{\to 0}} \right) = |x| = R$$

$\Rightarrow R = |x| < 1$ for absolute convergence

Check $R = |x| = 1 \Rightarrow x = \pm 1$

$$x = 1 \Rightarrow 1 + \sum_{n=1}^{\infty} \frac{(-1)^n (1)^n}{n^2 + 2n} = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 2n}$$

$$\lim_{n \to \infty} \frac{1}{n^2 + 2n} = 0 \quad , 0 < \frac{1}{(n+1)^2 + 2(n+1)} < \frac{1}{n^2 + 2n}$$

$\Rightarrow$ converges

$$x = -1 \Rightarrow 1 + \sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{n^2 + 2n} = 1 + \sum_{n=1}^{\infty} \frac{1}{n^2 + 2n} \leq 1 + \underbrace{\sum_{n=1}^{\infty} \frac{1}{n^2}}_{\text{converges}}$$

$\Rightarrow$ converges by ordinary comparison test

$\Rightarrow$ Convergence Set $= \boxed{-1 \leq x \leq 1}$

25) $1 + \frac{x+1}{2} + \frac{(x+1)^2}{2^2} + \frac{(x+1)^3}{2^3} + \cdots$

$$= \sum_{n=0}^{\infty} \frac{(x+1)^n}{2^n} , \quad |a_n| = \frac{|x+1|^n}{2^n} , \quad |a_{n+1}| = \frac{|x+1|^{n+1}}{2^{n+1}}$$

$$R = \lim_{n\to\infty} \frac{\frac{|x+1|^{n+1}}{2^{n+1}}}{\frac{|x+1|^n}{2^n}} = \lim_{n\to\infty} \frac{2^n |x+1|^{n+1}}{2^{n+1} |x+1|^n} = \frac{|x+1|}{2} = R$$

Converges for $R < 1 \Rightarrow \frac{|x+1|}{2} < 1 , \quad |x+1| < 2$

$$\Rightarrow \quad -3 < x < 1$$

Check $R = \frac{|x+1|}{2} = 1 \Rightarrow |x+1| = 2 \quad x = -3 , \quad x = 1$

$x = -3 , \quad \sum_{n=0}^{\infty} \frac{(-3+1)^n}{2^n} = \sum_{n=0}^{\infty} \frac{(-2)^n}{2^n} = \sum_{n=0}^{\infty} (-1)^n$ diverges

$x = 1 , \quad \sum_{n=0}^{\infty} \frac{(1+1)^n}{2^n} = \sum_{n=0}^{\infty} 1$ diverges

$\Rightarrow$ Convergence Set $= \boxed{-3 < x < 1}$