Assignment 5 Solutions

Section 7.4 : 1, 5, 14, 17, 21, 22

1) $\int x\sqrt{x+1}\,dx$

$x+1$ is linear

$u = \sqrt{x+1}$ $\qquad 2u\,du = dx$

$\Rightarrow u^2 = x+1$

$x = u^2 - 1$

$= \int (u^2-1)u\,(2u\,du)$

$= 2\int u^4 - u^2\,du = 2\left[\frac{1}{5}u^5 - \frac{1}{3}u^3\right] + C$

$= \frac{2}{5}u^5 - \frac{2}{3}u^3 + C$

$= \boxed{\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C}$

5) $\int_1^2 \frac{dt}{\sqrt{t}+e}$

t is linear

$u = \sqrt{t}$ $\qquad t=1 \Rightarrow u = \sqrt{1} = 1$

$u^2 = t$ $\qquad t=2 \Rightarrow u = \sqrt{2}$

$2u\,du = dt$

$= \int_1^{\sqrt{2}} \frac{2u\,du}{u+e}$

$= 2\int_1^{\sqrt{2}} \frac{u}{u+e}\,du$

Simplify with long division

$$\begin{array}{r} 1 \\ u+e\,\overline{)\,u+0} \\ -(u+e) \\ \hline -e \end{array} \Rightarrow \frac{u}{u+e} = 1 - \frac{e}{u+e}$$

$= 2\int_1^{\sqrt{2}} \left(1 - \frac{e}{u+e}\right) du$

$= 2\int_1^{\sqrt{2}} du - 2e\int_1^{\sqrt{2}} \frac{1}{u+e}\,du = 2\left[u\right]_1^{\sqrt{2}} - 2e\left[\ln(u+e)\right]_1^{\sqrt{2}}$

$$= 2(\sqrt{2}-1) - 2e\left(\ln(\sqrt{2}+e) - \ln(1+e)\right)$$ $\ast \ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$

$$= \boxed{2\sqrt{2} - 2 - 2e\ln\left(\frac{\sqrt{2}+e}{1+e}\right)}$$

14) $\int \frac{t}{\sqrt{1-t^2}}\,dt$

let $u = 1-t^2$

$du = -2t\,dt$

$\Rightarrow t\,dt = -\frac{1}{2}du$

$$= \int \frac{1}{\sqrt{u}}\left(\frac{-1}{2}du\right)$$

$$= -\frac{1}{2}\int u^{-1/2}\,du = \frac{-1}{2}\left[\frac{u^{1/2}}{1/2}\right] + C$$

$$= -\frac{1}{2}\left(2\sqrt{1-t^2}\right) + C$$

$$= \boxed{-\sqrt{1-t^2} + C}$$

17) $\int \frac{dx}{\sqrt{x^2+2x+5}}$

radical is of the form ax^2+bx+c

$\Rightarrow$ complete the square

$$= \int \frac{dx}{\sqrt{x^2+2x+4+1}} = \int \frac{dx}{\sqrt{(x+2)^2+1}}$$

let $u = x+2$

$du = dx$

$$= \int \frac{du}{\sqrt{u^2+1}}$$

now radical is of the form

$x^2+a^2 \Rightarrow$ trig substitute

let $u = a\tan(t) = \tan(t)$

$\Rightarrow \sqrt{u^2+1} = \sec(t)$

$u = \tan(t)$

$du = \sec^2(t)\,dt$

$$= \int \frac{\sec^2(t)\,dt}{\sec(t)}$$

$$= \int \sec(t)\,dt$$

$$= \ln|\sec(t) + \tan(t)| + C$$

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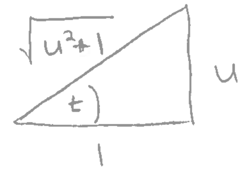
$= \ln|\sec(t) + \tan(t)| + C$



$\tan = \frac{\text{opposite}}{\text{adjacent}}$

$\sec(t) = \sqrt{u^2+1}$, $\tan(t) = u$

$= \ln|\sqrt{u^2+1} + u| + C$, $u = x+2$

$= \boxed{\ln\left|\sqrt{(x+2)^2+1} + x + 2\right| + C}$

21) $\int \sqrt{5-4x-x^2}\,dx = \int \sqrt{-(x^2+4x-5)}\,dx$

$= \int \sqrt{-(x^2+4x+4-9)}\,dx = \int \sqrt{9-(x+2)^2}\,dx$

let $u = x+2$ $\quad = \int \sqrt{9-u^2}\,du$

$du = dx$

radical is of the form $a^2-u^2 \Rightarrow a=3$ let $u = a\sin(t)$

$u = 3\sin(t)$

$\Rightarrow \sqrt{9-u^2} = 3\cos(t)$ $\quad du = 3\cos(t)\,dt$

$= \int (3\cos(t))(3\cos(t)\,dt) = 9\int \cos^2(t)\,dt$

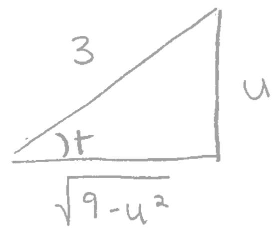
type 1 even from 7.3

$\cos^2(t) = \frac{1+\cos(2t)}{2}$ $\quad = 9\int \frac{1}{2} + \frac{1}{2}\cos(2t)\,dt$

$= \frac{9}{2}\int dt + \frac{9}{2}\int \cos(2t)\,dt$

$$= \frac{9}{2} t + \frac{9}{4} \sin(2t) + C$$

$u = 3\sin(t)$ double angle formula



$\sin(t) = \frac{u}{3}$

$\sin(2t) = 2\sin(t)\cos(t)$

$t = \sin^{-1}\left(\frac{u}{3}\right)$

$\cos(t) = \frac{\sqrt{9-u^2}}{3}$

$$= \frac{9}{2} t + \frac{9}{4} \sin(t)\cos(t) + C$$

$$= \frac{9}{2} \sin^{-1}\left(\frac{u}{3}\right) + \frac{9}{4}\left(\frac{u}{3}\right)\left(\frac{\sqrt{9-u^2}}{3}\right) + C$$

$$= \frac{9}{2} \sin^{-1}\left(\frac{x+2}{3}\right) + \frac{1}{4} u\sqrt{9-u^2} + C$$

$$= \boxed{\frac{9}{2} \sin^{-1}\left(\frac{x+2}{3}\right) + \frac{1}{4}(x+2)\sqrt{9-(x+2)^2} + C}$$

22) $\displaystyle\int \frac{dx}{\sqrt{16+6x-x^2}} = \int \frac{dx}{\sqrt{-(x^2-6x-16)}} = \int \frac{dx}{\sqrt{-(x^2-6x+9-25)}}$

$$= \int \frac{dx}{\sqrt{25-(x-3)^2}}$$

$u = x-3$

$du = dx$

$$= \int \frac{du}{\sqrt{25-u^2}}$$

$u = 5\sin(t)$

$du = 5\cos(t)\, dt$

$\sqrt{25-u^2} = 5\cos(t)$

$$= \int \frac{5\cos(t)}{5\cos(t)}\,dt = \int dt = t + C$$

$\sin(t) = \frac{u}{5}$ $\qquad = \sin^{-1}\left(\frac{u}{5}\right) + C$

$t = \sin^{-1}\left(\frac{u}{5}\right)$ $\qquad = \boxed{\sin^{-1}\left(\frac{x-3}{5}\right) + C}$

Section 7.5 : 1, 5, 14, 27, 30, 41

1) $\int \frac{1}{x(x+1)}\,dx$ $\qquad \frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$

$1 = A(x+1) + Bx$

$= \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$

$x = 0 \Rightarrow 1 = A$

$= \int \frac{1}{x}\,dx - \int \frac{1}{x+1}\,dx$ $\qquad x = -1 \Rightarrow 1 = B(-1) \Rightarrow B = -1$

$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$

$= \ln|x| - \ln|x+1| + C$

$= \boxed{\ln\left|\frac{x}{x+1}\right| + C}$

5) $\int \frac{x-11}{x^2+3x-4}$ $\qquad \frac{x-11}{x^2+3x-4} = \frac{x-11}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$

$x - 11 = A(x-1) + B(x+4)$

$x = 1 \Rightarrow 1 - 11 = B(5)$

$-10 = 5B \Rightarrow B = -2$

$x = -4 \Rightarrow -4 - 11 = A(-5)$

$-15 = -5A \Rightarrow A = 3$

$$\frac{x-11}{x^2+3x-4} = \frac{3}{x+4} - \frac{2}{x-1}$$

$$= \int \frac{3}{x+4} - \frac{2}{x-1}\, dx$$

$$= \boxed{3\ln|x+4| - 2\ln|x-1| + C}$$

14) $$\int \frac{7x^2+2x-3}{(2x-1)(3x+2)(x-3)}\, dx$$

$$\frac{7x^2+2x-3}{(2x-1)(3x+2)(x-3)} = \frac{A}{2x-1} + \frac{B}{3x+2} + \frac{C}{x-3}$$

$$\Rightarrow 7x^2+2x-3 = A(3x+2)(x-3) + B(2x-1)(x-3) + C(2x-1)(3x+2)$$

$x = 3 \Rightarrow$ $$7(3)^2 + 2(3) - 3 = C(5)(11)$$

$$66 = 55C \quad \Rightarrow \quad C = \frac{6}{5}$$

$x = \frac{1}{2} \Rightarrow$ $$7\left(\tfrac{1}{2}\right)^2 + 2\left(\tfrac{1}{2}\right) - 3 = A\left(3\cdot\tfrac{1}{2}+2\right)\left(\tfrac{1}{2}-3\right)$$

$$\frac{7}{4} + 1 - 3 = A\left(\frac{7}{2}\right)\left(\frac{-5}{2}\right)$$

$$-\frac{1}{4} = -\frac{35}{4}A \quad \Rightarrow \quad A = \frac{1}{35}$$

$x = \frac{-2}{3}$ $$7\left(\tfrac{-2}{3}\right)^2 + 2\left(\tfrac{-2}{3}\right) - 3 = B\left(2\left(\tfrac{-2}{3}\right)-1\right)\left(\tfrac{-2}{3}-3\right)$$

$$\frac{28}{9} - \frac{12}{9} - \frac{27}{9} = B\left(\frac{-4}{3} - \frac{3}{3}\right)\left(\frac{-2}{3} - \frac{9}{3}\right)$$

$$-\frac{11}{9} = B\left(\frac{-7}{3}\right)\left(\frac{-11}{3}\right) = \frac{77}{9}B$$

$$B = \frac{-1}{7}$$

$$\Rightarrow \int \frac{1/35}{2x-1}\,dx + \int \frac{-1/7\, dx}{3x+2} + \int \frac{6/5}{x-3}\, dx$$

$u = 2x-1$ $\quad$ $v = 3x+2$

$du = 2\,dx$ $\quad$ $dv = 3\,dx$

$$= \frac{1}{70}\int \frac{du}{u} - \frac{1}{21}\int \frac{dv}{v} + \frac{6}{5}\int \frac{1}{x-3}\, dx$$

$$= \frac{1}{70}\ln|2x-1| - \frac{1}{21}\ln|3x+2| + \frac{6}{5}\ln|x-3| + C$$

27) $\displaystyle\int \frac{2x^2+x-8}{x^3+4x}\,dx = \int \frac{2x^2+x-8}{x(x^2+4)}\,dx$

$$\frac{2x^2+x-8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$\Rightarrow \quad 2x^2+x-8 = A(x^2+4) + x(Bx+C)$$

$$x=0 \quad \Rightarrow \quad -8 = A(4) \qquad A=-2$$

$$\Rightarrow \quad 2x^2+x-8 = -2(x^2+4) + Bx^2 + Cx$$

$$\Rightarrow \quad Bx^2+Cx = 4x^2+x$$

$$\Rightarrow \quad B=4,\ C=1$$

$$\Rightarrow \quad \frac{2x^2+x-8}{x(x^2+4)} = \frac{-2}{x} + \frac{4x+1}{x^2+4}$$

$$= \int \frac{-2}{x}\,dx + \int \frac{4x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx$$

$u = x^2+4$ $\qquad$ $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$ form

$= -2\ln|x|$ $\qquad$ $du = 2x\,dx$

$$= -2\ln|x| + 2\int \frac{du}{u} + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \boxed{-2\ln|x| + 2\ln|x^2+4| + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C}$$

30) $\displaystyle\int \frac{1}{x^4-16}\,dx = \int \frac{1}{(x^2+4)(x^2-4)}\,dx = \int \frac{1}{(x^2+4)(x+2)(x-2)}\,dx$ 8

$$\frac{1}{x^4-16} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{Cx+D}{x^2+4}$$

$$\Rightarrow \quad 1 = A(x-2)(x^2+4) + B(x+2)(x^2+4) + (Cx+D)(x+2)(x-2)$$

$$x=2 \quad \Rightarrow \quad 1 = B(4)(4+4) \quad \Rightarrow \quad B = 1/32$$

$$x=-2 \quad \Rightarrow \quad 1 = A(-4)(8) \quad \Rightarrow \quad A = -1/32$$

$$x=0 \quad \Rightarrow \quad 1 = \frac{-1}{32}(-2)(4) + \frac{1}{32}(2)(4) + D(2)(-2)$$

$$1 = \frac{1}{4} + \frac{1}{4} - 4D \quad \Rightarrow \quad -4D = \frac{1}{2}$$

$$D = -\frac{1}{8}$$

$$x=1 \quad \Rightarrow \quad 1 = \frac{-1}{32}(-1)(5) + \frac{1}{32}(3)(5) + \left(C + \frac{-1}{8}\right)(3)(-1)$$

$$1 = \frac{5}{32} + \frac{15}{32} - 3C + \frac{3}{8}$$

$$1 = \frac{20}{32} + \frac{12}{32} - 3C \quad \Rightarrow \quad 1 = 1 - 3C$$

$$C = 0$$

$$\Rightarrow \int \frac{-1/32}{x+2}\,dx + \int \frac{1/32}{x-2}\,dx + \int \frac{-1/8}{x^2+4}$$

$\tan^{-1}$ form

$$= \frac{-1}{32}\ln|x+2| + \frac{1}{32}\ln|x-2| - \frac{1}{8}\left(\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right) + C$$

$$= \boxed{\frac{1}{32}\left(\ln|x-2| - \ln|x+2| - 2\tan^{-1}\left(\frac{x}{2}\right)\right) + C}$$

41) $y' = y(1-y) \qquad y(0) = \frac{1}{2}$

$$\frac{dy}{y(1-y)} = dt$$

$$\frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{1-y} = \frac{1}{y} + \frac{1}{1-y} = \frac{1}{y} - \frac{1}{y-1}$$

$$1 = A(1-y) + B(y)$$

$$y = 1 \Rightarrow \quad 1 = B$$

$$y = 0 \Rightarrow \quad 1 = A$$

$$\int \frac{1}{y} - \frac{1}{y-1}\, dy = \int dt$$

$$\ln|y| - \ln|y-1| = t + C$$

$$\ln\left|\frac{y}{y-1}\right| = t + C$$

$$\frac{y}{y-1} = e^{t+C} = Ce^{t}$$

$$y(0) = \frac{1}{2} \quad \Rightarrow \quad \frac{1/2}{\frac{1}{2}-1} = Ce^{0} = C$$

$$\frac{\frac{1}{2}}{-\frac{1}{2}} = C \quad \Rightarrow \quad C = -1$$

$$\frac{y}{y-1} = -e^{t} \quad \Rightarrow \quad y = -(y-1)e^{t}$$

$$y = -ye^{t} + e^{t}$$

$$y + ye^{t} = e^{t}$$

$$y(1+e^{t}) = e^{t}$$

$$\boxed{y = \frac{e^{t}}{1+e^{t}}, \quad y(3) = \frac{e^{3}}{1+e^{3}}}$$

Section 7.6 : 1, 3, 9, 10

1) $\int x e^{-5x}\, dx$

u substitution will fail,
product of functions $\Rightarrow$ try integration by parts

$u = x \qquad dv = e^{-5x}\, dx$

$du = dx \qquad v = \frac{-1}{5} e^{-5x}$

$$\Rightarrow \int x e^{-5x}\, dx = \frac{-1}{5} x e^{-5x} - \int \left(\frac{-1}{5} e^{-5x}\right) dx$$

$$= \frac{-1}{5} x e^{-5x} + \int \frac{1}{5} e^{-5x}\, dx$$

$$= \frac{-1}{5} x e^{-5x} + \frac{1}{5}\left[\frac{-1}{5} e^{-5x}\right]$$

$$= \boxed{\frac{-1}{5} x e^{-5x} - \frac{1}{25} e^{-5x} + C}$$

$$= \frac{-1}{5} e^{-5x}\left(x + \frac{1}{5}\right) + C$$

3) $\int_1^2 \frac{\ln(x)}{x}\, dx$

$\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x} \Rightarrow$ try u substitution

$u = \ln(x) \qquad x = 1 \Rightarrow u = \ln(1) = 0$

$du = \frac{1}{x} dx \qquad x = 2 \Rightarrow u = \ln(2)$

$$= \int_0^{\ln(2)} u\, du = \left[\frac{1}{2} u^2\right]_0^{\ln(2)}$$

$$= \frac{1}{2} \ln(2)^2 - \frac{1}{2}(0)^2 = \boxed{\frac{1}{2}(\ln(2))^2}$$

9) $\displaystyle\int_0^5 x\sqrt{x+2}\,dx$ linear radical form

$u=\sqrt{x+2}$

$u^2 = x+2 \Rightarrow x = u^2-2$

$2u\,du = dx$

$x=0 \Rightarrow u=\sqrt{2}$

$x=5 \Rightarrow u=\sqrt{7}$

$$= \int_{\sqrt{2}}^{\sqrt{7}} (u^2-2)u\,(2u\,du)$$

$$= \int_{\sqrt{2}}^{\sqrt{7}} (2u^4-4u^2)\,du = \left[\frac{2}{5}u^5 - \frac{4}{3}u^3\right]_{\sqrt{2}}^{\sqrt{7}}$$

$$= \frac{2}{5}(7)^{5/2} - \frac{4}{3}(7)^{3/2} - \left(\frac{2}{5}(2)^{5/2} - \frac{4}{3}(2)^{3/2}\right)$$

$$= \frac{2\cdot 7^2}{5}(7)^{1/2} - \frac{4\cdot 7}{3}(7)^{1/2} - \frac{2\cdot 2^2}{5}(2)^{1/2} + \frac{4\cdot 2}{3}(2)^{1/2}$$

$$= \frac{2\cdot 3\cdot 7^2 - 4\cdot 7\cdot 5}{15}\sqrt{7} - \frac{2^3\cdot 3 - 2^3\cdot 5}{15}\sqrt{2}$$

$$= \frac{14(21-10)}{15}\sqrt{7} - \frac{8(3-5)}{15}\sqrt{2}$$

$$= \boxed{\frac{2}{15}\left(77\sqrt{7}+8\sqrt{2}\right)}$$

10) $\displaystyle\int_3^4 \frac{1}{t-\sqrt{2t}}\,dt$ linear radical

$\Rightarrow u=\sqrt{2t}$

$u^2 = 2t$ $\quad t = \frac{u^2}{2}$

$2u\,du = 2\,dt$

$u\,du = dt$

$t=3 \Rightarrow u=\sqrt{6}$

$t=4 \Rightarrow u=\sqrt{8}$

$$= \int_{\sqrt{6}}^{\sqrt{8}} \frac{u\,du}{\frac{u^2}{2}-u}$$

$$= \int_{\sqrt{6}}^{\sqrt{8}} \frac{\cancel{u}\, du}{\cancel{u}\left(\frac{1}{2}u - 1\right)} = \int_{\sqrt{6}}^{\sqrt{8}} \frac{du}{\frac{1}{2}u - 1} = \int_{\sqrt{6}}^{\sqrt{8}} \frac{2\, du}{u - 2}$$

$$= 2 \int_{\sqrt{6}}^{\sqrt{8}} \frac{du}{u-2} = 2\Big[\ln|u-2|\Big]_{\sqrt{6}}^{\sqrt{8}}$$

$$= \boxed{2\left(\ln|\sqrt{8}-2| - \ln|\sqrt{6}-2|\right)}$$