Assignment 4 Solutions

Section 7.1: 1, 5, 10, 12, 25, 28, 37, 44, 51

1) $\int (x-2)^5\,dx$ let $u = x-2$, $du = dx$

$$= \int u^5\,du = \frac{1}{6}u^6 + C = \boxed{\frac{1}{6}(x-2)^6 + C}$$

5) $\int \frac{dx}{x^2+4}$ standard form for $\tan^{-1}$

$$= \int \frac{dx}{x^2+2^2} = \boxed{\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C}$$

10) $\int \frac{5}{\sqrt{2t+1}}\,dt$ let $u = 2t+1$, $du = 2\,dt$

$$= \frac{1}{2}\int \frac{5}{\sqrt{2t+1}}\,2\,dt = \frac{5}{2}\int \frac{du}{\sqrt{u}} = \frac{5}{2}\int u^{-1/2}\,du$$

$$= \frac{5}{2}\left(2u^{1/2}\right) + C = 5u^{1/2} + C$$

$$= \boxed{5(2t+1)^{1/2} + C}$$

12) $\int e^{\cos(z)}\sin(z)\,dz$ let $u = \cos(z)$, $du = -\sin(z)\,dz$

$$= -\int e^{\cos(z)}(-\sin(z)\,dz) = -\int e^u\,du = -e^u + C$$

$$= \boxed{-e^{\cos(z)} + C}$$

25) $\displaystyle\int_0^1 t \cdot 3^{t^2}\, dt$ let $u = t^2$, $du = 2t\, dt$ $\Rightarrow$ $t=0 \Rightarrow u=0$, $t=1 \Rightarrow u=1$

$$= \frac{1}{2}\int_0^1 2t \cdot 3^{t^2}\, dt = \frac{1}{2}\int_0^1 3^u\, du = \frac{1}{2}\left[\frac{3^u}{\ln(3)}\right]_0^1$$

$$= \frac{1}{2}\left(\frac{3^1}{\ln(3)} - \frac{3^0}{\ln(3)}\right) = \frac{1}{2\ln(3)}(3-1) = \frac{2}{2\ln(3)} = \boxed{\frac{1}{\ln(3)}}$$

28) $\displaystyle\int \frac{\sin(4t-1)}{1-\sin^2(4t-1)}\, dt = \int \frac{\sin(4t-1)}{\cos^2(4t-1)}\, dt$ $\Rightarrow$ $u = \cos(4t-1)$, $du = -4\sin(4t-1)\, dt$

pythagorean identity
$\sin^2(x) + \cos^2(x) = 1$

$$= -\frac{1}{4}\int \frac{-4\sin(4t-1)dt}{(\cos(4t-1))^2} = -\frac{1}{4}\int \frac{du}{u^2} = -\frac{1}{4}\int u^{-2}\, du$$

$$= -\frac{1}{4}\left(-u^{-1}\right) + C = \frac{1}{4}u^{-1} + C$$

$$= \frac{1}{4\cos(4t-1)} + C = \boxed{\frac{1}{4}\sec(4t-1) + C}$$

37) $\displaystyle\int \frac{e^{\tan^{-1}(2t)}}{1+4t^2}\, dt$, let $u = \tan^{-1}(2t)$, $du = 2\dfrac{1}{1+(2t)^2}\, dt = \dfrac{2}{1+4t^2}\, dt$

$$= \frac{1}{2}\int e^u\, du = e^u + C = \boxed{\frac{1}{2}e^{\tan^{-1}(2t)} + C}$$

44) $$\int \frac{dt}{2t\sqrt{4t^2-1}}$$ looks like $\sec^{-1}$ form

let $u = 2t$

$du = 2dt$

$$= \frac{1}{2}\int \frac{2\,dt}{2t\sqrt{4t^2-1}} = \frac{1}{2}\int \frac{du}{u\sqrt{u^2-1}} = \frac{1}{2}\sec^{-1}(u) + C$$

$$= \boxed{\frac{1}{2}\sec^{-1}(2t) + C}$$

51) $$\int \frac{x+1}{9x^2+18x+10}\,dx$$, let $u = 9x^2 + 18x + 10$

$du = (18x + 18)dx = 18(x+1)\,dx$

$$= \frac{1}{18}\int \frac{18(x+1)}{9x^2+18x+10}\,dx = \frac{1}{18}\int \frac{du}{u} = \frac{1}{18}\ln|u| + C$$

$$= \boxed{\frac{1}{18}\ln|9x^2 + 18x + 10| + C}$$

Section 7.2: 1, 11, 16, 25, 37, 41

1) $$\int xe^x\,dx$$

$u = x$ $\quad dv = e^x\,dx$

$du = dx$ $\quad v = e^x$

$$= [xe^x] - \int e^x\,dx = \boxed{xe^x - e^x + C}$$

11) $\int \ln(3x)\,dx$ $\quad u = \ln(3x) \quad dv = dx$

$du = \frac{1}{3x} \cdot 3\,dx \quad v = x$

$$= \left[x \ln(3x)\right] - \int x \cdot \frac{1}{3x} \cdot 3\,dx$$

$$= x\ln(3x) - \int dx = x\ln(3x) - x = \boxed{x\left(\ln(3x) - 1\right)}$$

16) $\int_2^3 \frac{\ln(2x^5)}{x^2}\,dx$ $\quad u = \ln(2x^5) \quad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{2x^5} \cdot 10x^4\,dx \quad v = \int x^{-2}\,dx = -x^{-1}$

$$= \frac{-\ln(2x^5)}{x} - \int -x^{-1}(5x^{-1})\,dx = \frac{-\ln(2x^5)}{x} + \int 5x^{-2}\,dx$$

$$= \frac{-\ln(2x^5)}{x} + 5(-x^{-1}) + C$$

$$= \boxed{\frac{-1}{x}\left(\ln(2x^5) + 5\right) + C}$$

25) $\int x^5 \sqrt{x^3+4}\,dx$ $\quad$ if we let $u = \sqrt{x^3+4}$ the integral gets worse

$\Rightarrow$ let $u = \frac{1}{3}x^3 \quad dv = 3x^2\sqrt{x^3+4}$

$du = x^2\,dx \quad v = \int 3x^2\sqrt{x^3+4}$ $\quad$ let $z = x^3+4$

$dz = 3x^2\,dx$

$$= \int z^{1/2}\,dz = \frac{2}{3}z^{3/2} = \frac{2}{3}(x^3+4)^{3/2}$$

$$= \left[\frac{1}{3}x^3 \cdot \frac{2}{3}(x^3+4)^{3/2}\right] - \int \frac{2}{3}(x^3+4)^{3/2} x^2\,dx$$

$$= \frac{2}{9}x^3(x^3+4)^{3/2} - \frac{1}{3}\int \frac{2}{3}(x^3+4)^{3/2} \cdot 3x^2\,dx$$

let $u = x^3+4$

$du = 3x^2\,dx$

$$= \frac{2}{9}x^3(x^3+4)^{3/2} - \frac{2}{9}\int u^{3/2}\,du$$

$$= \frac{2}{9}x^3(x^3+4)^{3/2} - \frac{2}{9}\left(\frac{2}{5}u^{5/2}\right) + C$$

$$= \frac{2}{9}x^3(x^3+4)^{3/2} - \frac{2}{45}(x^3+4)^{5/2} + C$$

$$= \frac{2}{9}(x^3+4)^{3/2}\left(x^3 - \frac{2}{5}(x^3+4)\right) + C$$

$$= \boxed{\frac{2}{9}(x^3+4)^{3/2}\left(\frac{3}{5}x^3 - \frac{8}{5}\right) + C}$$

37) $\int x^2 e^x\,dx$

$u = x^2 \qquad dv = e^x\,dx$

$du = 2x\,dx \qquad v = e^x$

$$= x^2e^x - \int 2xe^x\,dx$$

$u = 2x \qquad dv = e^x\,dx$

$du = 2\,dx \qquad v = e^x$

$$= x^2e^x - \left[(2xe^x) - \int 2e^x\,dx\right]$$

$$= x^2e^x - 2xe^x + 2\int e^x\,dx$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

$$= \boxed{e^x(x^2-2x+2) + C}$$

41) $\int e^t \cos(t)\, dt$

$u = \cos(t) \qquad dv = e^t\, dt$

$du = -\sin(t)\, dt \qquad v = e^t$

$$\Rightarrow \int e^t \cos(t)\, dt = e^t \cos(t) + \int e^t \sin(t)\, dt$$

$u = \sin(t) \qquad dv = e^t\, dt$

$du = \cos(t)\, dt \qquad v = e^t$

$$\Rightarrow \int e^t \cos(t)\, dt = e^t \cos(t) + e^t \sin(t) - \int e^t \cos(t)\, dt$$

$$\Rightarrow 2\int e^t \cos(t)\, dt = e^t \cos(t) + e^t \sin(t) + C$$

$$\rightarrow \int e^t \cos(t)\, dt = \boxed{\frac{e^t}{2}\big(\cos(t) + \sin(t)\big) + C}$$

Section 7.3 : 3, 7, 12, 13, 21, 27

3) $\int \sin^3(x)\, dx$ Type 1 odd

$$= \int \sin^2(x) \sin(x)\, dx = \int \big(1 - \cos^2(x)\big) \sin(x)\, dx$$

$$= \int \sin(x)\, dx - \int \cos^2(x) \sin(x)\, dx$$

$u = \cos(x)$

$du = -\sin(x)\, dx$

$$= \underbrace{\int \sin(x)\, dx}_{\text{standard form}} + \int \underbrace{\cos^2(x)}_{u^2} \underbrace{(-\sin(x)\, dx)}_{du}$$

$$= -\cos(x) + \int u^2\, du + C = -\cos(x) + \frac{1}{3}u^3 + C$$

$$= \boxed{-\cos(x) + \frac{1}{3}\cos^3(x) + C}$$

7) $\int \sin^5(4x)\cos^2(4x)\,dx$ Type 2 odd

$$= \int \sin^4(4x)\cos^2(4x)\sin(4x)\,dx$$

$$= \int \left(1-\cos^2(4x)\right)^2 \cos^2(4x)\sin(4x)\,dx$$

$$= \int \left(1 - 2\cos^2(4x) + \cos^4(4x)\right)\cos^2(4x)\sin(4x)\,dx$$

$$= \int \left(\cos^2(4x) - 2\cos^4(4x) + \cos^6(4x)\right)\sin(4x)\,dx$$

let $u = \cos(4x)$

$du = -4\sin(4x)\,dx$

$$= \frac{-1}{4}\int \left(u^2 - 2u^4 + u^6\right)du$$

$$= \frac{-1}{4}\left(\frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7\right) + C$$

$$= \boxed{-\frac{1}{12}\cos^3(4x) + \frac{1}{10}\cos^5(4x) - \frac{1}{28}\cos^7(4x) + C}$$

12) $\int \cos^6(\theta)\sin^2(\theta)\,d\theta$ Type 2 even

$$= \int \left(\frac{1+\cos(2\theta)}{2}\right)^3 \left(\frac{1-\cos(2\theta)}{2}\right) d\theta$$

$$= \frac{1}{16}\int (1+\cos(2\theta))(1-\cos(2\theta))(1+\cos(2\theta))^2\,d\theta$$

$$= \frac{1}{16}\int (1-\cos^2(2\theta))(1+\cos(2\theta))(1+\cos(2\theta))\,d\theta$$

$$= \frac{1}{16}\int (1+\cos(2\theta)-\cos^2(2\theta)-\cos^3(2\theta))(1+\cos(2\theta))\,d\theta$$

$$= \frac{1}{16}\int (1+2\cos(2\theta)-\underbrace{2\cos^3(2\theta)}_{\text{type 1 odd}}-\underbrace{\cos^4(2\theta)}_{\text{type 1 even}})\,d\theta$$

$$= \frac{1}{16}\int d\theta + \frac{1}{8}\int \cos(2\theta)\,d\theta - \frac{1}{8}\int \cos^3(2\theta)\,d\theta - \frac{1}{16}\int \cos^4(2\theta)\,d\theta$$

$u = 2\theta$
$du = 2\,d\theta$

$$= \frac{1}{16}\theta + \frac{1}{16}\sin(2\theta) - \frac{1}{8}\int (1-\sin^2(2\theta))\cos(2\theta)\,d\theta$$

$$- \frac{1}{16}\int \left(\frac{1+\cos(4\theta)}{2}\right)^2 d\theta$$

$$= \frac{1}{16}(\theta+\sin(2\theta)) - \frac{1}{8}\int \cos(2\theta)\,d\theta + \frac{1}{8}\int \sin^2(2\theta)\cos(2\theta)\,d\theta$$

$$- \frac{1}{64}\int (1+2\cos(4\theta)+\cos^2(4\theta))\,d\theta$$

$$= \frac{1}{16}\left(\theta + \cancel{\sin(2\theta)}\right) - \cancel{\frac{1}{16}\sin(2\theta)} + \frac{1}{8}\int \sin^2(2\theta)\cos(2\theta)\,d\theta$$

$u = \sin(2\theta)$

$du = 2\cos(2\theta)\,d\theta$

$$-\frac{1}{64}\int \left(1 + 2\cos(4\theta) + \cos^2(4\theta)\right)d\theta$$

$$= \frac{1}{16}\theta + \frac{1}{16}\int u^2\,d\theta - \frac{1}{64}\int d\theta - \frac{1}{32}\int \cos(4\theta)\,d\theta - \frac{1}{64}\int \cos^2(4\theta)\,d\theta$$

$$= \frac{1}{16}\theta + \frac{1}{48}\sin^3(2\theta) - \frac{1}{64}\theta - \frac{1}{128}\sin(4\theta) - \frac{1}{64}\int\left(\frac{1+\cos(8\theta)}{2}\right)d\theta$$

$$= \frac{4}{64}\theta + \frac{1}{48}\sin^3(2\theta) - \frac{1}{64}\theta - \frac{1}{128}\sin(4\theta) - \frac{1}{128}\int d\theta - \frac{1}{128}\int \cos(8\theta)\,d\theta$$

$$= \frac{3}{64}\theta + \frac{1}{48}\sin^3(2\theta) - \frac{1}{128}\sin(4\theta) - \frac{1}{128}\theta - \frac{1}{956}\sin(8\theta) + C$$

$$= \boxed{\frac{5}{128}\theta + \frac{1}{48}\sin^3(\theta) - \frac{1}{128}\sin(4\theta) - \frac{1}{956}\sin(8\theta) + C}$$

13) $\int \sin(4y)\cos(5y)\,dy$ Type 3

$$= \int \frac{1}{2}\left[\sin(4+5)y + \sin(4-5)y\right]dy$$

$$= \frac{1}{2}\left[\int \sin(9y)\,dy + \int \sin(-y)\,dy\right.$$

$$= \frac{1}{2}\left[-\cos(9y)\cdot\frac{1}{9} + \left(-\cos(-y)\right)\cdot\frac{1}{-1}\right] + C$$

$$= -\frac{1}{18}\cos(9y) + \frac{1}{2}\cos(-y) + C$$

note: $\cos(-x) = \cos(x)$ b/c even function

$$= \boxed{\frac{1}{2}\cos(y) - \frac{1}{18}\cos(9y) + C}$$

21) $\int \tan^3(x)\,dx$ Type. 4 odd

$$= \int \tan^2(x)\tan(x)\,dx = \int \left(\sec^2(x)-1\right)\tan(x)\,dx$$

$$= \int \sec^2(x)\tan(x)\,dx - \int \tan(x)\,dx$$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$\downarrow$ standard form

$$= \int u\,du - \left(-\ln|\cos(x)|\right) + C$$

$$= \frac{u^2}{2} + \ln|\cos(x)| + C$$

$$= \boxed{\frac{1}{2}\tan^2(x) + \ln|\cos(x)| + C}$$

27) $\int \tan^3(x)\sec^2(x)\,dx$ Type 5, m odd / n even

note: we can u sub right now

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$$= \int u^3\,du = \frac{1}{4}u^4 + C = \boxed{\frac{1}{4}\tan^4(x) + C}$$