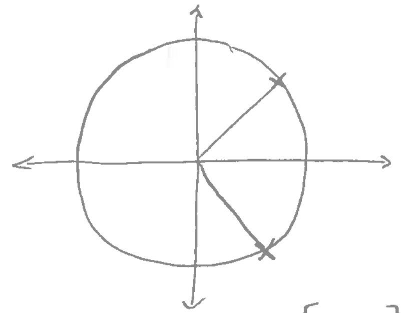
Assignment 3 Solutions

Section 6.8: 1, 5, 9, 19, 22, 39, 45, 49, 55, 63, 70, 72

1) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = x$

$\Rightarrow \frac{\sqrt{2}}{2} = \cos(x)$

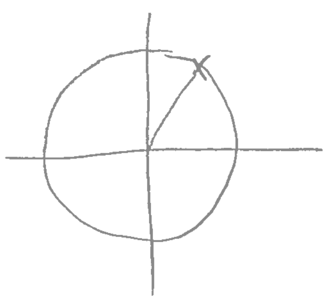


$x = \frac{\pi}{4}, \frac{-\pi}{4}$, but domain of $\cos(x) = [0, \pi]$

$\Rightarrow \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$

5) $\tan^{-1}(\sqrt{3}) = y$

$\Rightarrow \sqrt{3} = \tan(y) = \frac{\sin(y)}{\cos(y)} \Longrightarrow$ $\sin(y) = \frac{\sqrt{3}}{2}$, $\cos(y) = \frac{1}{2}$



$\Rightarrow y = \boxed{\frac{\pi}{3}}$

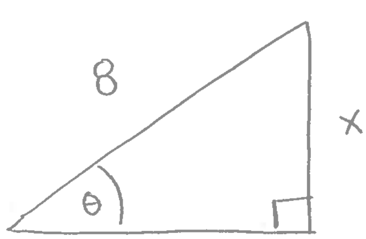
9) $\sin\left(\sin^{-1}(0.4567)\right)$

domain of $\sin^{-1}(x) = [-1, 1]$

$\Rightarrow 0.4567 \in [-1, 1]$

$\Rightarrow \sin\left(\sin^{-1}(0.4567)\right) = \boxed{0.4567}$

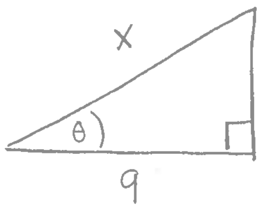
19)



$\sin(\theta) = \frac{x}{8} \Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{x}{8}\right)}$

$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$

22)



$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\Rightarrow \cos(\theta) = \frac{9}{x} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{9}{x}\right)}$$

39) $y = \ln(2 + \sin(x))$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln(2 + \sin(x))\right] \quad \text{chain rule}$$

$$= \frac{1}{2 + \sin(x)}\left(\frac{d}{dx}\left[2 + \sin(x)\right]\right)$$

$$= \boxed{\frac{\cos(x)}{2 + \sin(x)}}$$

45) $y = x^3 \tan^{-1}(e^x)$

$$\frac{dy}{dx} = D_x\left[x^3 \tan^{-1}(e^x)\right] \quad \text{product rule}$$

chain rule

let $u = e^x$

$\frac{du}{dx} = e^x$

$$= D_x\left[x^3\right] \tan^{-1}(e^x) + x^3 D_x\left[\tan^{-1}(e^x)\right]$$

$$= 3x^2 \tan^{-1}(e^x) + x^3 D_u\left[\tan^{-1}(u)\right] D_x\left[u\right]$$

$$= 3x^2 \tan^{-1}(e^x) + x^3 \left(\frac{1}{1 + u^2}\right) \frac{du}{dx}$$

$$= \boxed{3x^2 \tan^{-1}(e^x) + \frac{x^3 e^x}{1 + e^{2x}}}$$

49) $y = \sec^{-1}(x^3)$

Chain rule
let $u = x^3$

$$\frac{dy}{dx} = D_x\left[\sec^{-1}(x^3)\right]$$

$$= \frac{d}{du}\left[\sec^{-1}(u)\right]\frac{du}{dx}$$

$$= \frac{1}{|u|\sqrt{u^2-1}} \frac{d}{dx}\left[x^3\right]$$

$$= \frac{1}{|x^3|\sqrt{x^6-1}}(3x^2) = \frac{3x^2}{|x^3|\sqrt{x^6-1}} = \boxed{\frac{3}{|x|\sqrt{x^6-1}}}$$

55) $\int \cos(3x)\,dx$

let $u = 3x$ $\Rightarrow$ $dx = \frac{du}{3}$
$du = 3dx$

$$\int \cos(u)\frac{du}{3} = \frac{1}{3}\int \cos(u)\,du$$

$$= \frac{1}{3}\left[\sin(u)\right] + C$$

$$= \boxed{\frac{1}{3}\sin(3x) + C}$$

63) $$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \left[\tan^{-1}(x)\right]_{-1}^{1} = \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \frac{\pi}{4} - \frac{-\pi}{4} = \boxed{\frac{\pi}{2}}$$

70) $$\int \frac{1}{2x^2+8x+25}\,dx$$

$$= \int \frac{1}{2\left(x^2+4x+\frac{25}{2}\right)}\,dx = \frac{1}{2}\int \frac{1}{x^2+4x+\frac{25}{2}}\,dx$$

complete the square:

$$= \frac{1}{2}\int \frac{1}{\underbrace{x^2+4x+4}_{(x+2)^2}\,\underbrace{-4}_{\text{add 0 creatively}}+\frac{25}{2}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{(x+2)^2+\frac{25}{2}-4}\,dx = \frac{1}{2}\int \frac{1}{(x+2)^2+\frac{17}{2}}\,dx$$

now u substitute, let $u = x+2$
$du = dx$

$$= \frac{1}{2}\int \frac{1}{u^2+\frac{17}{2}}\,du = \frac{1}{2}\int \frac{1}{u^2+\left(\sqrt{\frac{17}{2}}\right)^2}\,du$$

now our integral is in $\tan^{-1}$ standard form

note: $$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

$$\Rightarrow \frac{1}{2}\int \frac{1}{u^2+\left(\sqrt{\frac{17}{2}}\right)^2}\,du = \frac{1}{2}\frac{1}{\sqrt{\frac{17}{2}}}\tan^{-1}\left(\frac{u}{\sqrt{\frac{17}{2}}}\right)+C$$

$$= \boxed{\frac{\sqrt{2}}{2\sqrt{17}}\tan^{-1}\left(\frac{\sqrt{2}(x+2)}{\sqrt{17}}\right)+C}$$

72) $\displaystyle\int \frac{x+1}{\sqrt{4-9x^2}}\,dx$ note $x+1$ is not $\frac{d}{dx}\left[4-9x^2\right]$

$\Rightarrow$ try splitting the integral up

$$= \int \frac{x}{\sqrt{4-9x^2}}\,dx + \int \frac{1}{\sqrt{4-9x^2}}\,dx$$

u substitution

$u = 4-9x^2$

$du = -18x\,dx$

trig inverse form

let $v = 3x$

$dv = 3\,dx$

$$= \frac{-1}{18}\int \frac{-18x\,dx}{\sqrt{4-9x^2}} + \frac{1}{3}\int \frac{3\,dx}{\sqrt{4-9x^2}}$$

$$= \frac{-1}{18}\int \frac{du}{\sqrt{u}} + \frac{1}{3}\int \frac{dv}{\sqrt{4-v^2}}$$

standard form for $\sin^{-1}\left(\frac{x}{a}\right)$

$a = 2$

$$= \frac{-1}{18}\int u^{-1/2}\,du + \frac{1}{3}\sin^{-1}\left(\frac{v}{2}\right) + C$$

$$= \frac{-1}{18}\left(2u^{1/2}\right) + \frac{1}{3}\sin^{-1}\left(\frac{v}{2}\right) + C$$

$$= \boxed{\frac{-1}{9}\sqrt{4-9x^2} + \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C}$$

Section 6.9 : 13, 16, 38, 43

13) $y = \sinh^2(x) = (\sinh(x))^2$ chain rule

$D_x[y] = 2\sinh(x)\, D_x[\sinh(x)]$

$= \boxed{2\sinh(x)\cosh(x)}$

16) $y = \cosh^3(x) = (\cosh(x))^3$ chain rule

$D_x[y] = 3(\cosh(x))^2\, D_x[\cosh(x)]$

$= \boxed{3\cosh^2(x)\sinh(x)}$

38) $\int \sinh(3x+2)\, dx$ let $u = 3x+2$

$du = 3\,dx$

$= \frac{1}{3}\int \sinh(3x+2)\, 3\,dx = \frac{1}{3}\int \sinh(u)\, du$

$= \frac{1}{3}\cosh(u) + C$

$= \boxed{\frac{1}{3}\cosh(3x+2) + C}$

43) $\int \cos(x)\sinh(\sin(x))\, dx$ let $u = \sin(x)$

$du = \cos(x)\, dx$

$= \int \sinh(u)\, du = \cosh(u) + C$

$= \boxed{\cosh(\sin(x)) + C}$