6.1: 3, 5, 8, 13, 15, 22, 23, 31

3) $D_x\left[\ln(x^2+3x+\pi)\right]$ Chain rule outside = $\ln(\ )$

u = inside = $x^2+3x+\pi$

$= D_u\left[\ln(u)\right]\cdot\frac{du}{dx}$

$= \frac{1}{u}\cdot(2x+3) = \frac{1}{x^2+3x+\pi}(2x+3)$

$= \boxed{\frac{2x+3}{x^2+3x+\pi}}$

5) $D_x\left[\ln(x-4)^3\right] = D_x\left[3\ln(x-4)\right]$

$= 3D_x\left[\ln(x-4)\right]$ Chain Rule $(u = x-4)$

$= 3D_u\left[\ln(u)\right]\frac{du}{dx} = 3\cdot\frac{1}{x-4}\cdot 1$

$= \boxed{\frac{3}{x-4}}$

8) $\frac{dy}{dx}$ if $y = x^2\ln x$

$\frac{dy}{dx} = \frac{d}{dx}\left[x^2\ln(x)\right]$ product rule

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^2\right]\ln(x) + x^2 \frac{d}{dx}\left[\ln(x)\right]$$

$$= 2x\ln(x) + x^2 \cdot \frac{1}{x}$$

$$= 2x\ln(x) + x = \boxed{x\left(2\ln(x)+1\right)}$$

13) $f'(81)$ if $f(x) = \ln\left(\sqrt[3]{x}\right)$

$$f'(x) = \frac{d}{dx}\left[\ln(x)^{1/3}\right] = \frac{d}{dx}\left[\frac{1}{3}\ln(x)\right]$$

$$= \frac{1}{3}\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{3}\cdot\frac{1}{x} = \frac{1}{3x}$$

$$\Rightarrow f'(81) = \frac{1}{3\cdot 81} = \frac{1}{243} = \frac{1}{3^5} = \boxed{3^{-5}}$$

15) $\displaystyle\int \frac{1}{2x+1}\,dx$ let $u = 2x+1$, $du = 2\,dx$ $\Rightarrow dx = \frac{1}{2}du$

$$= \int \frac{1}{u}\cdot\frac{1}{2}du = \frac{1}{2}\int\frac{1}{u}du = \frac{1}{2}\ln|u| + C$$

$$= \boxed{\frac{1}{2}\ln|2x+1| + C}$$

22) $\displaystyle\int_0^1 \frac{t+1}{2t^2+4t+3}\,dt$ let $u = 2t^2+4t+3$

$du = (4t+4)\,dt$

$\Rightarrow dt = \dfrac{du}{4(t+1)}$

$$\int_{t=0}^{t=1} \frac{\cancel{t+1}}{2t^2+4t+3}\left(\frac{du}{4\cancel{(t+1)}}\right)$$

$u = 2t^2 + 4t + 3$

when $t=0$ $u=3$

$t=1$ $u = 2+4+3 = 9$

$$= \frac{1}{4}\int_3^9 \frac{1}{u}\,du = \frac{1}{4}\Big[\ln|u|\Big]_3^9$$

$$= \frac{1}{4}\left(\ln|9| - \ln|3|\right)$$

$$= \frac{1}{4}\left(2\ln(3) - \ln(3)\right) = \boxed{\frac{\ln(3)}{4}}$$

23) $\displaystyle\int \frac{x^2}{x-1}\,dx$ need to use polynomial long division

$$\begin{array}{r} x + 1 + \frac{1}{x-1} \\ x-1 \,\big)\, \overline{x^2 + 0x + 0} \\ -(x^2 - x) \\ \hline x + 0 \\ -(x-1) \\ \hline 1 \end{array}$$

$$= \int x + 1 + \frac{1}{x-1}\,dx$$

$$= \int x\,dx + \int 1\,dx + \int \frac{1}{x-1}\,dx$$

$$= \frac{1}{2}x^2 + x + C + \int \frac{1}{x-1}\,dx$$

let $u = x-1$

$du = dx$

$$= \frac{1}{2}x^2 + x + C + \int \frac{1}{u}\,du$$

$$= \frac{1}{2}x^2 + x + \ln|u| + C$$

$$= \boxed{\frac{1}{2}x^2 + x + \ln|x-1| + C}$$

31) $y = \dfrac{x+11}{\sqrt{x^3-4}} \Rightarrow \ln(y) = \ln\left(\dfrac{x+11}{(x^3-4)^{1/2}}\right)$

$$\ln(y) = \ln(x+11) - \ln\left((x^3-4)^{1/2}\right)$$

$$\ln(y) = \ln(x+11) - \frac{1}{2}\ln(x^3-4)$$

$$\Rightarrow D_x\left[\ln(y)\right] = D_x\left[\ln(x+11)\right] - \frac{1}{2}D_x\left[\ln(x^3-4)\right] \longleftarrow \text{chain rule}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x+11} - \frac{1}{2}\cdot\frac{1}{x^3-4}\cdot 3x^2$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x+11}\cdot\frac{2(x^3-4)}{2(x^3-4)} - \frac{3x^2}{2(x^3-4)}\frac{(x+11)}{(x+11)}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2(x^3-4) - 3x^2(x+11)}{2(x^3-4)(x+11)}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x^3-8-3x^3-33x^2}{2(x^3-4)(x+11)}$$

$$\frac{dy}{dx} = \frac{-x^3-33x^2-8}{2(x^3-4)\cancel{(x+11)}}\cdot\underbrace{\left(\frac{\cancel{x+11}}{(x^3-4)^{1/2}}\right)}_{y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x^3-33x^2-8}{2(x^3-4)^{3/2}}}$$

6.2 : 7, 8, 15, 19, 32, 37

7) $f(x) = -x^5 - x^3$

$\Rightarrow f'(x) = -5x^4 - 3x^2 < 0$ for all x

$\Rightarrow f(x)$ is strictly monotonic and $f^{-1}(x)$ exists.

8) $f(x) = x^7 + x^5$

$\Rightarrow f'(x) = 7x^6 + 5x^4 > 0$ for all x

$\Rightarrow f(x)$ is strictly monotonic and $f^{-1}(x)$ exists.

15) $f(x) = x + 1 = y$

$\Rightarrow x = y - 1 = f^{-1}(y)$

$\Rightarrow \boxed{f^{-1}(x) = x - 1}$

$f^{-1}(f(x)) = f^{-1}(x+1) = (x+1) - 1 = x$ ✓

$f(f^{-1}(x)) = f(x-1) = (x-1) + 1 = x$ ✓

19) $f(x) = -\frac{1}{x-3} = y$

$-1 = y(x-3) \Rightarrow x - 3 = \frac{-1}{y}$

$x = 3 - \frac{1}{y} = f^{-1}(y)$

$\Rightarrow \boxed{f^{-1}(x) = 3 - x^{-1}}$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{-1}{x-3}\right) = 3 - \left(\frac{-1}{x-3}\right)^{-1} = 3 - \frac{3-x}{1} = x \checkmark$$ (6)

$$f(f^{-1}(x)) = f(3 - x^{-1}) = -\frac{1}{(3-x^{-1})-3} = -\frac{1}{-x^{-1}} = x \checkmark$$

32) $f(x) = x^2 - 3x + 1$

$f'(x) = 2x - 3 \quad \Rightarrow \quad f'(x) > 0 \quad$ for $x > \frac{3}{2}$

restricted domain $= \left(\frac{3}{2}, \infty\right)$

find $f^{-1}(x) \quad \Rightarrow \quad f(x) = x^2 - 3x + 1 = y$ — complete the square

$$x^2 - 3x + \frac{9}{4} = y - 1 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = y + \frac{5}{4}$$

$$\left(x - \frac{3}{2}\right) = \pm\sqrt{y + \frac{5}{4}}$$

$$x = \frac{3}{2} \pm \sqrt{y + \frac{5}{4}} = f^{-1}(y)$$

$$\Rightarrow \quad f^{-1}(x) = \frac{3}{2} \pm \sqrt{x + \frac{5}{4}}$$

when $x = 2, \quad f(x) = 4 - 6 + 1 = -1$

$\Rightarrow$ when $x = -1 \quad f^{-1}(x) = 2 = \frac{3}{2} \pm \sqrt{-1 + \frac{5}{4}}$

$$2 = \frac{3}{2} \pm \sqrt{\frac{1}{4}} = \frac{3}{2} \pm \frac{1}{2}$$

$\Rightarrow$ we need the positive root

$$\Rightarrow \quad \boxed{f^{-1}(x) = \frac{3}{2} + \sqrt{x + \frac{5}{4}}}$$

37) find $\left[f^{-1}(2)\right]'$ $\Rightarrow$ $y = 2 = f(x)$

$$f(x) = 3x^5 + x - 2 = 2$$

$$\Rightarrow \quad x = 1$$

By theorem B we have

$$(f^{-1})'(y=2) = \frac{1}{f'(x=1)}$$

$$f'(x) = 15x^4 + 1 \quad \Rightarrow \quad f'(1) = 15(1)^4 + 1 = 16$$

$$\Rightarrow \boxed{(f^{-1})'(2) = \frac{1}{16}}$$

Section 6.3: 3, 7, 12, 16, 27, 37, 40

3) $e^{3\ln(x)} = e^{\ln(x^3)} = \boxed{x^3}$

7) $\ln(x^3 e^{-3x}) = \ln(x^3) + \ln(e^{-3x})$

$$= \boxed{3\ln(x) - 3x}$$

12) $y = e^{\sqrt{x+2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}\left[e^{\sqrt{x+2}}\right]$ let $u = \sqrt{x+2}$

chain rule $= \dfrac{d}{du}\left[e^u\right] \dfrac{du}{dx}$

$$= e^u \cdot \frac{d}{dx}\left[(x+2)^{1/2}\right]$$

$$= e^{\sqrt{x+2}} \cdot \frac{1}{2}(x+2)^{-1/2}$$

$$= \boxed{\frac{e^{\sqrt{x+2}}}{2\sqrt{x+2}}}$$

16) $y = e^{x/\ln(x)}$

$D_x[y] = D_x\left[e^{x/\ln(x)}\right]$ chain rule

$= e^{x/\ln(x)} \cdot D_x\left[\frac{x}{\ln(x)}\right]$

$= e^{\frac{x}{\ln(x)}} \cdot \left(\frac{D_x[x]\ln(x) - x\, D_x[\ln(x)]}{(\ln(x))^2}\right)$

$= e^{\frac{x}{\ln(x)}} \left(\frac{\ln(x) - 1}{\ln(x)^2}\right)$

27) $f(x) = xe^{-x}$

e^{-x} is defined for all real x, as is x

$\Rightarrow$ xe^{-x} has domain $= \mathbb{R}$

$f'(x) = D_x\left[xe^{-x}\right] = e^{-x} + x(-1)e^{-x}$

$= e^{-x} - xe^{-x}$

$e^{-x} > 0$ for all x $\Rightarrow$ $f'(x) > 0$ when $x < 1$ increasing

$f'(x) < 0$ when $x > 1$ decreasing

$f'(x) = 0 = e^{-x} - xe^{-x} = e^{-x}(1-x)$

$\Rightarrow$ $f'(x) = 0$ when $x = 1$ local max

$f''(x) = D_x\left[e^{-x} - xe^{-x}\right]$

$= -e^{-x} - \left(D_x[x]e^{-x} + x\, D_x[e^{-x}]\right)$

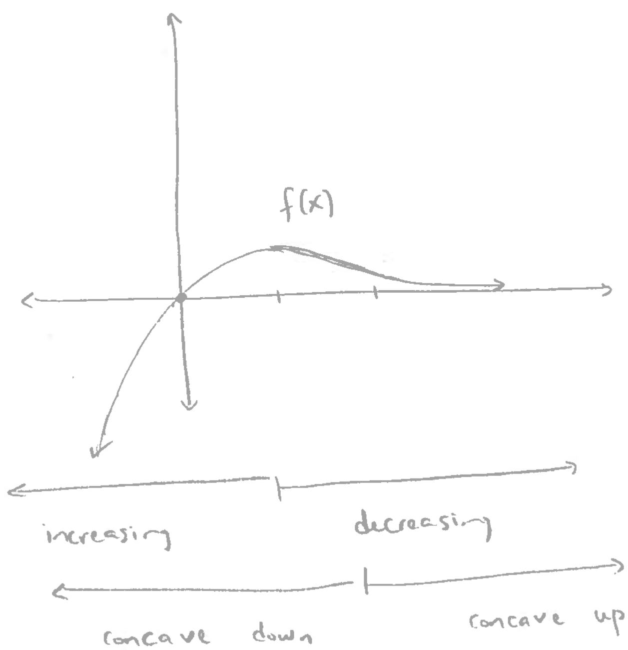
$$f''(x) = -e^{-x} - \left(e^{-x} - xe^{-x}\right)$$

$$= -2e^{-x} + xe^{-x} = e^{-x}(x-2)$$

$\Rightarrow f''(x) = 0 \quad \Rightarrow \quad x = 2$ inflection point

$f''(x) > 0$ for $x > 2$ concave up

$f''(x) < 0$ for $x < 2$ concave down



$f(1) = 1e^{-1} = \frac{1}{e} \approx$

$f(0) = 0$

37) $\int e^{3x+1}\, dx$ let $u = 3x+1$ $\quad dx = \frac{1}{3} du$

$du = 3\, dx$

$$= \int e^u \left(\tfrac{1}{3} du\right) = \frac{1}{3}\int e^u\, du = \frac{1}{3}e^u + C$$

$$= \boxed{\frac{1}{3}e^{3x+1} + C}$$

40) $\displaystyle\int \frac{e^x}{e^x - 1}\,dx$ let $u = e^x - 1$, $du = e^x\,dx$, $dx = \frac{1}{e^x}\,du$

$$= \int \frac{\cancel{e^x}}{u}\left(\frac{1}{\cancel{e^x}}\,du\right)$$

$$= \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|e^x - 1| + C}$$

Section 6.4 : 1, 5, 17, 18, 31, 32

1) $\log_2(8) = x$

$$x = \log_2(8) = \log_2(2^3) = 3\log_2(2) = \boxed{3}$$

5) $2\log_9\left(\frac{x}{3}\right) = 1$

$$\log_9\left(\left(\frac{x}{3}\right)^2\right) = 1$$

$$\log_9\left(\frac{x^2}{9}\right) = 1 \Rightarrow \log_9(x^2) - \log_9(9) = 1$$

$$\log_9(x^2) - 1 = 1$$

$$2\log_9(x) = 2$$

$$\log_9(x) = 1$$

$$x = 9^1 = \boxed{9}$$

17) $D_x\left[6^{2x}\right] = D_x\left[e^{\ln(6^{2x})}\right]$

$= D_x\left[e^{2x\ln(6)}\right] = e^{2x\ln(6)}\, D_x\left[2x\ln(6)\right]$

$= 2\ln(6)\, e^{2x\ln(6)} = \boxed{2\ln(6)\, 6^{2x}}$

18) $D_x\left[3^{2x^2-3x}\right]$

$= D_x\left[e^{\ln(3^{2x^2-3x})}\right] = D_x\left[e^{(2x^2-3x)\ln(3)}\right]$

$= e^{(2x^3-3x)\ln(3)}\, D_x\left[(2x^2-3x)\ln(3)\right]$

$= \boxed{(4x-3)\ln(3)\, 3^{2x^2-3x}}$

31) $y = (x^2+1)^{\ln(x)}$

$\ln(y) = \ln\left[(x^2+1)^{\ln(x)}\right] = \ln(x)\ln(x^2+1)$

$\Rightarrow D_x\left[\ln(y)\right] = D_x\left[\ln(x)\ln(x^2+1)\right]$

$\frac{1}{y}\frac{dy}{dx} = D_x\left[\ln(x)\right]\ln(x^2+1) + D_x\left[\ln(x^2+1)\right]\ln(x)$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{\ln(x^2+1)}{x} + \frac{\ln(x)}{x^2+1}\cdot 2x$

$$\frac{dy}{dx} = \left(\frac{\ln(x^2+1)}{x} + \frac{2x\ln(x)}{x^2+1}\right) y$$

$$= \boxed{\left(\frac{\ln(x^2+1)}{x} + \frac{2x\ln(x)}{x^2+1}\right)(x^2+1)^{\ln(x)}}$$

32) $y = \left(\ln(x^2)\right)^{2x+3}$

$$\Rightarrow \ln(y) = \ln\left(\left(\ln x^2\right)^{2x+3}\right) = (2x+3)\ln\left(\ln(x^2)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = D_x\left[(2x+3)\ln\left(2\ln(x)\right)\right]$$

$$= D_x(2x+3)\ln\left(2\ln(x)\right) + (2x+3)D_x\left[\ln\left(2\ln(x)\right)\right]$$

$$= 2\ln\left(2\ln(x)\right) + (2x+3)\frac{1}{2\ln(x)} \cdot D_x\left[2\ln(x)\right]$$

$$= 2\ln\left(2\ln(x)\right) + (2x+3)\cdot\frac{1}{\cancel{2}\ln(x)}\cdot\frac{\cancel{2}}{x}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \ln(x^2)^{2x+3}\left(2\ln\left(2\ln(x)\right) + \frac{2x+3}{x\ln(x)}\right)}$$