Homework 10 Solutions

Section 10.5: 3, 9, 11, 13, 17, 18

3) ① $(3, 2\pi)$

② $(-2, \frac{1}{3}\pi)$

③ $(-2, \frac{-1}{4}\pi)$

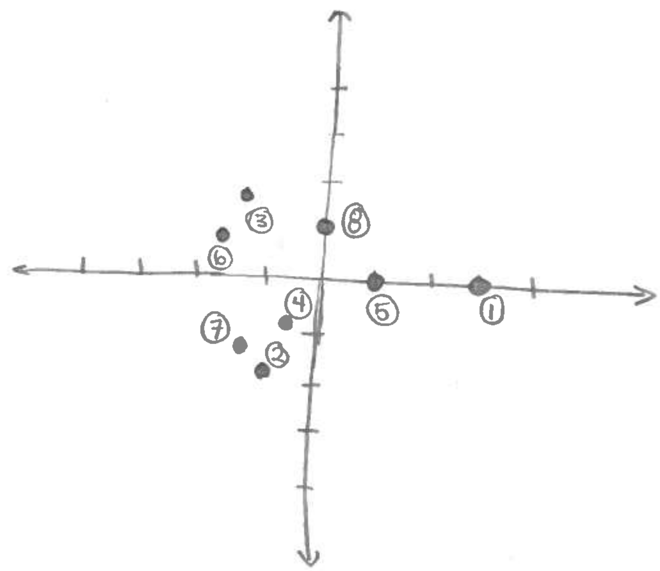
④ $(-1, 1)$

⑤ $(1, -4\pi)$

⑥ $(\sqrt{3}, -\frac{7}{6}\pi)$

⑦ $(-2, \frac{1}{4}\pi)$

⑧ $(-1, \frac{-1}{2}\pi)$



9) a) $(x,y) = (3\sqrt{3}, 3)$ $\quad 1^{st}$ quadrant

$x = 3\sqrt{3}$

$y = 3$

$$r^2 = x^2 + y^2 = (3\sqrt{3})^2 + (3)^2 = 27 + 9$$

$$r^2 = 36, \quad r = 6$$

$$\tan(\theta) = \frac{y}{x} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \Rightarrow \sin(\theta) = \frac{1}{2}, \ \cos(\theta) = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6} \Rightarrow \boxed{\left(6, \frac{\pi}{6}\right) = (r, \theta)}$$

b) $(x,y) = (-2\sqrt{3}, 2)$ $\quad 2^{nd}$ quadrant

$x = -2\sqrt{3}$

$y = 2$

$$r^2 = x^2 + y^2 = (-2\sqrt{3})^2 + (2)^2 = 12 + 4 = 16$$

$$r = 4$$

$$\tan(\theta) = \frac{y}{x} = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}} \Rightarrow -\frac{\pi}{6} = \theta \quad 4^{th} \text{ quadrant} \Rightarrow \text{add } \pi \text{ to get to second}$$

$$\theta = -\frac{\pi}{6} + \pi = \boxed{\frac{5\pi}{6}} \qquad \boxed{\left(4, \frac{5\pi}{6}\right)}$$

c) $(x,y) = (-\sqrt{2}, -\sqrt{2})$

$x = -\sqrt{2}$
$y = -\sqrt{2}$ $\Rightarrow$ 3^{rd} quadrant

$r^2 = (-\sqrt{2})^2 + (-\sqrt{2})^2 = 2 + 2 = 4$

$r = 2$

$\tan(\theta) = \dfrac{-\sqrt{2}}{-\sqrt{2}} = 1 \Rightarrow \theta = \dfrac{\pi}{4}$ 1^{st} quadrant $\Rightarrow$ add π

$\theta = \dfrac{\pi}{4} + \pi = \dfrac{5\pi}{4}$ $\boxed{\left(2, \frac{5\pi}{4}\right)}$

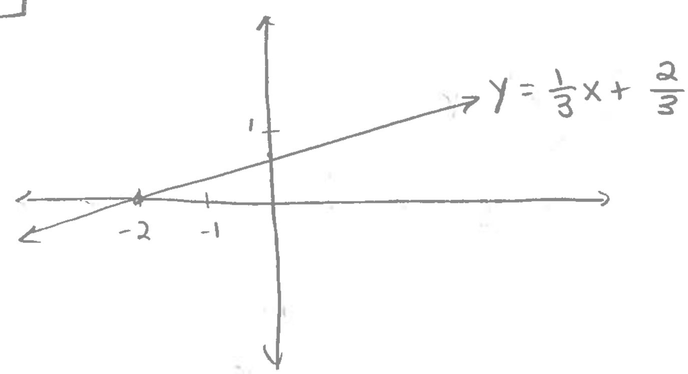
d) $(x,y) = (0,0)$ origen

$\Rightarrow$ $\boxed{r = 0, \ \theta = \text{anything}}$

11) $x - 3y + 2 = 0$

$3y = x + 2$

$y = \dfrac{1}{3}x + \dfrac{2}{3}$ = line



$y = r\sin\theta$
$x = r\cos\theta$ $\Rightarrow$ $r\sin\theta = \dfrac{1}{3}r\cos(\theta) + \dfrac{2}{3}$

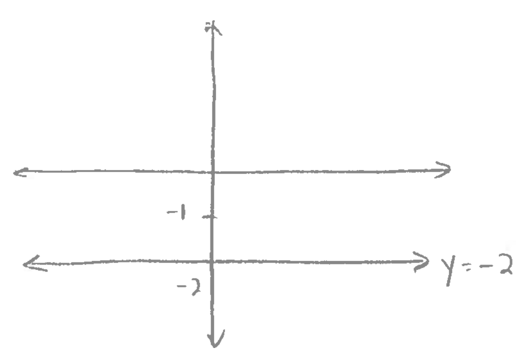
$r\left(\sin\theta - \dfrac{1}{3}\cos\theta\right) = \dfrac{2}{3}$

$r = \dfrac{2}{3\sin\theta - \cos\theta}$

13) $y = -2$

$\Rightarrow$ $r\sin(\theta) = -2$

$\boxed{r = \dfrac{-2}{\sin\theta}}$



17) $\theta = \frac{1}{2}\pi \Rightarrow \tan(\theta) = \tan\left(\frac{1}{2}\pi\right)$

$\tan(\theta) = \frac{y}{x} \Rightarrow \frac{y}{x} = \tan\left(\frac{1}{2}\pi\right) =$ undefined, division by 0

$$\Rightarrow \boxed{x = 0}$$

18) $r = 3 \Rightarrow r^2 = 3^2 \Rightarrow x^2 + y^2 = 3^2 = 9$

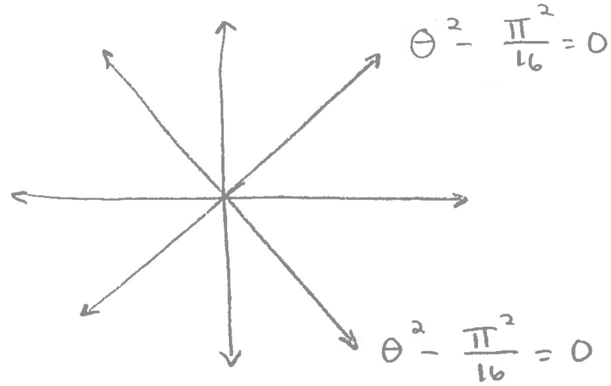
$r^2 = x^2 + y^2$

$$\boxed{x^2 + y^2 = 9}$$

Section 10.6 : 1, 3, 6, 9, 11, 14, 21, 33

1) $\theta^2 - \frac{\pi^2}{16} = 0$

$\Rightarrow \theta = \pm \frac{\pi}{4}$



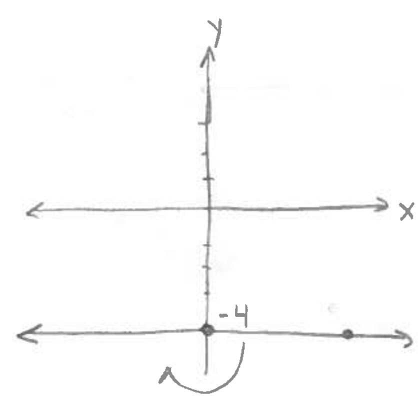
$\theta = -\theta$ for all r

$\Rightarrow$ x and y symmetry

3) $r \sin\theta + 4 = 0$

$r = \frac{-4}{\sin\theta}$

θ	r
$-\frac{\pi}{2}$	4
$-\frac{\pi}{4}$	$4\sqrt{2}$
0	$-\infty$
$\frac{\pi}{4}$	$-4\sqrt{2}$
$\frac{\pi}{2}$	-4



$(r, -\theta) \Rightarrow r = \frac{-4}{\sin(-\theta)} = \frac{4}{\sin(\theta)} = -r$

$(-r, -\theta) \Rightarrow -r = \frac{-4}{\sin(-\theta)} = \frac{4}{\sin(\theta)} = -r$ ✓ $\Rightarrow$ y-symmetric

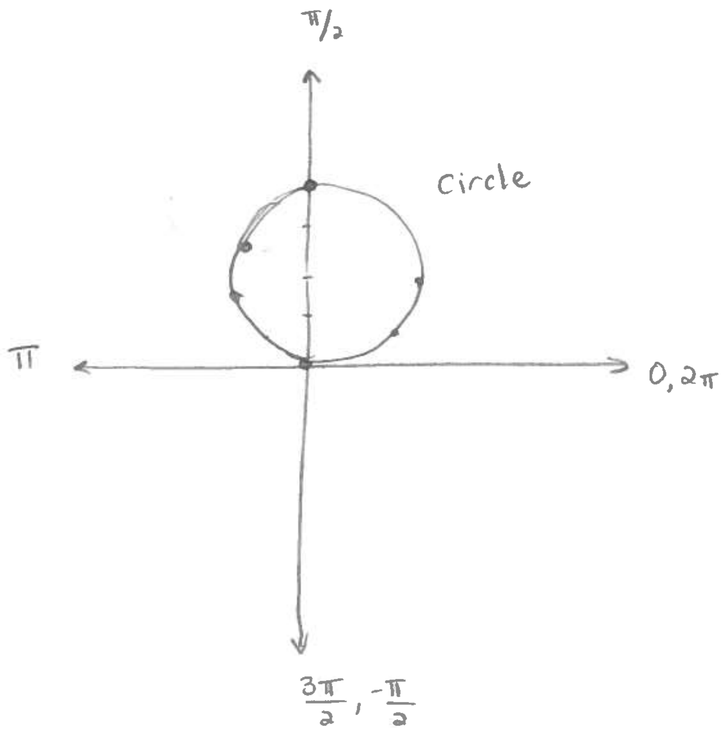
symmetric about y-axis

6) $r = 4\sin\theta$

$(r, -\theta) \Rightarrow \quad r = 4\sin(-\theta) = -4\sin\theta = -r$

$(-r, -\theta) \Rightarrow \quad -r = 4\sin(-\theta) = -4\sin\theta = -r$ ✓ $\Rightarrow$ y-axis symmetric

θ	r
$-\frac{\pi}{2}$	-4
$-\frac{\pi}{3}$	$-2\sqrt{3}$
$-\frac{\pi}{4}$	$-2\sqrt{2}$
0	0
$\frac{\pi}{4}$	$2\sqrt{2}$
$\frac{\pi}{6}$	2
$\frac{\pi}{2}$	4



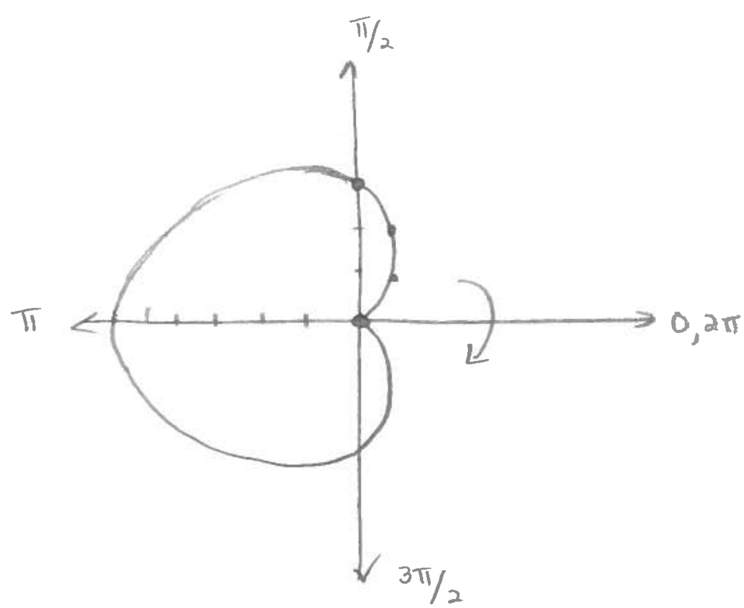
9) $r = 3 - 3\cos\theta$ (cardiod)

$(r, -\theta) \Rightarrow \quad r = 3 - 3\cos(-\theta) = 3 - 3\cos(\theta) = r$ ✓

x-symmetric $\Rightarrow$ let $\theta \in [0, \pi]$

$(-r, -\theta) \Rightarrow \quad -r = 3 - 3\cos(-\theta) = 3 - 3\cos\theta = r$ ✗

θ	$\cos\theta$	$r = 3 - 3\cos\theta$
0	1	0
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$3 - \frac{3\sqrt{2}}{2} = 3\left(1 - \frac{\sqrt{2}}{2}\right)$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{3}{2}$
$\frac{\pi}{2}$	0	3
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{9}{2}$
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$3\left(1 + \frac{\sqrt{2}}{2}\right)$
π	-1	6



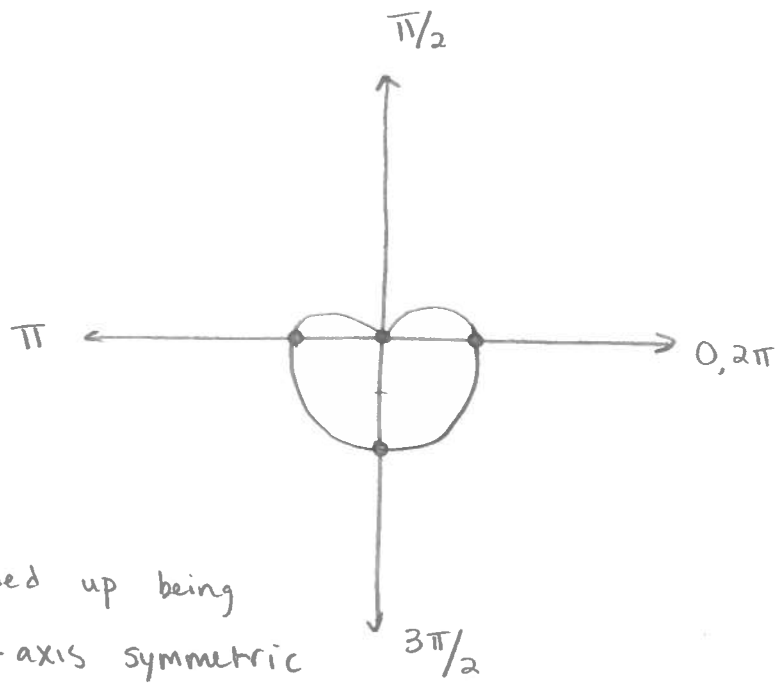
11) $r = 1 - \sin\theta$ (cardiod)

$(r, -\theta) \Rightarrow \quad r = 1 - \sin(-\theta) = 1 + \sin\theta \neq r$

$\Rightarrow$ no symmetry

$(-r, -\theta) \Rightarrow \quad -r = 1 - \sin(-\theta) = 1 + \sin\theta \neq -r \qquad \theta \in [0, 2\pi]$

θ	$\sin\theta$	r
0	0	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$1 - \frac{\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2} > 0$
$\frac{\pi}{2}$	1	0
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{2-\sqrt{2}}{2}$
π	0	1
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{2+\sqrt{2}}{2} > 1$
$\frac{3\pi}{2}$	-1	2
$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{2+\sqrt{2}}{2}$
2π	0	1



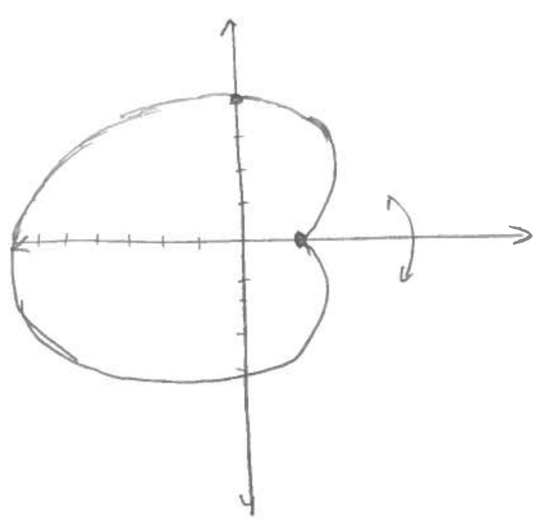
ended up being
y-axis symmetric
even though test failed

14) $r = 4 - 3\cos(\theta)$ limaçon

$(r, -\theta) \Rightarrow \quad r = 4 - 3\cos(-\theta) = 4 - 3\cos\theta = r$ ✓

x-axis symmetric $\Rightarrow \theta \in [0, \pi]$

θ	$\cos\theta$	r
0	1	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$4 - \frac{3\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{5}{2}$
$\frac{\pi}{2}$	0	4
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{11}{2}$
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$4 + \frac{3\sqrt{2}}{2}$
π	-1	7



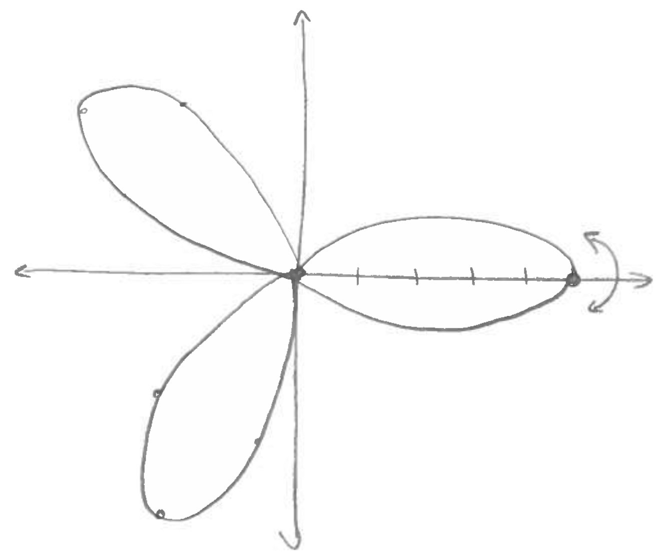
21) $r = 5\cos(3\theta)$ (3 leaved rose)

$(r, -\theta) \Rightarrow \quad r = 5\cos(-3\theta) = 5\cos(3\theta) = r$ ✓

x-axis symetric

$\theta \in [0, \pi]$

θ	$\cos(3\theta)$	r
0	1	5
$\frac{\pi}{6}$	0	0
$\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{5\sqrt{2}}{2}$
$\frac{\pi}{3}$	-1	-5
$\frac{\pi}{2}$	0	0
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{5\sqrt{2}}{2}$



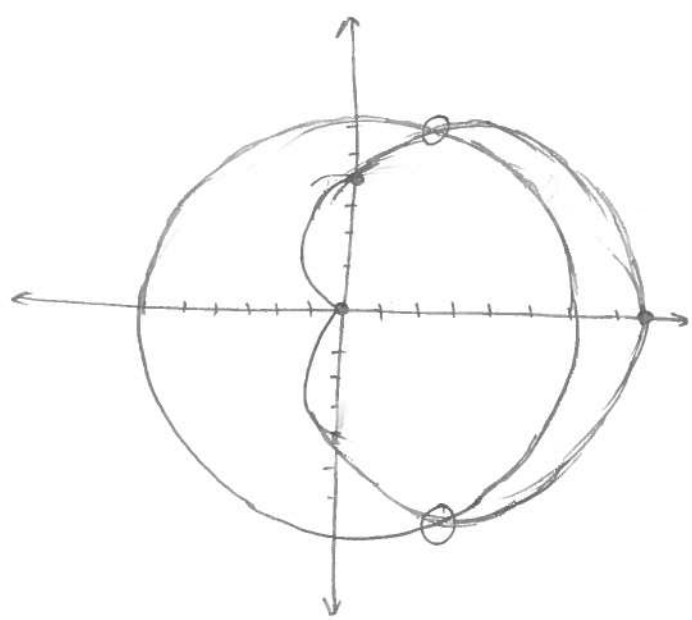
33) $r = 6, \quad r = 4 + 4\cos\theta$

circle radius = 6

cardioid

Cardioid:

θ	r
0	8
$\frac{\pi}{2}$	4
π	0
$\frac{3\pi}{2}$	4



Intersect when $r = 4 + 4\cos\theta = r = 6$

$4 + 4\cos\theta = 6$

$4\cos\theta = 2$

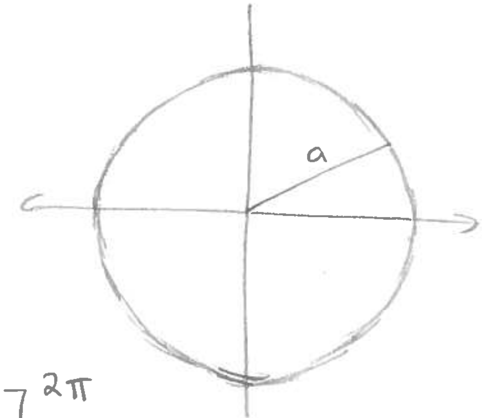
$\cos\theta = \frac{1}{2} \Rightarrow \theta = \pm\frac{\pi}{3}$

$4 + 4\cos\left(\frac{\pi}{3}\right) = 4 + 2 = 6$

$\Rightarrow \boxed{\left(6, \frac{\pi}{3}\right), \left(6, -\frac{\pi}{3}\right)}$

Section 10.7 : 1, 3, 6, 11, 15, 25

1) $r = a$ (circle of radius a)



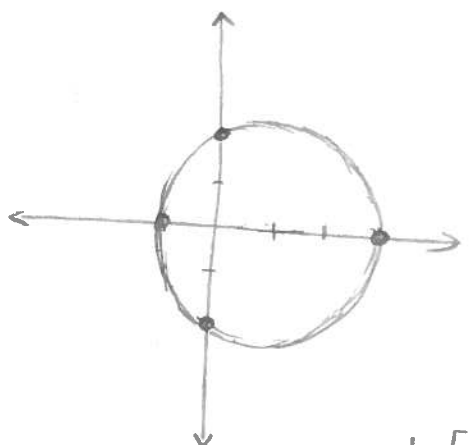
$$A = \frac{1}{2}\int_0^{2\pi} a^2 \, d\theta$$

$$= \frac{1}{2}(a^2)\int_0^{2\pi} d\theta = \frac{1}{2}a^2\Big[\theta\Big]_0^{2\pi} = \frac{1}{2}a^2(2\pi - 0)$$

$$= \boxed{\pi a^2}$$

3) $r = 2 + \cos(\theta)$ limaçon $a > b$

θ	r
0	3
$\frac{\pi}{2}$	2
π	1
$\frac{3\pi}{2}$	2
2π	3



$$A = \frac{1}{2}\int_0^{2\pi} (2 + \cos\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} 4 + 4\cos\theta + \cos^2\theta \, d\theta$$

type 1 even

$$= \frac{1}{2}\Big[4\theta\Big]_0^{2\pi} + \frac{1}{2}\Big[4\sin\theta\Big]_0^{2\pi} + \frac{1}{2}\int_0^{2\pi} \frac{1 + \cos(2\theta)}{2} \, d\theta$$

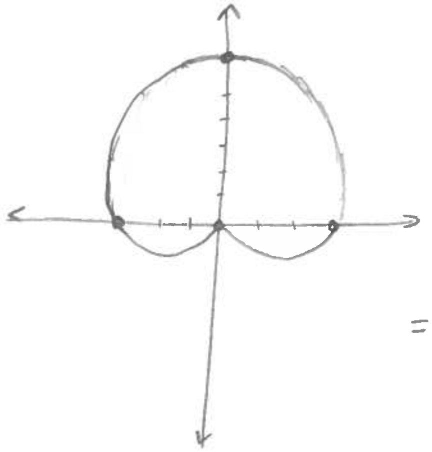
$$= \frac{1}{2}(8\pi - 0) + \frac{1}{2}\left(\cancelto{0}{4\sin(2\pi)} - \cancelto{0}{4\sin(0)}\right) + \frac{1}{4}\int_0^{2\pi} d\theta + \frac{1}{4}\int_0^{2\pi} \cos(2\theta)\, d\theta$$

$$= 4\pi + \frac{1}{4}(2\pi - 0) + \cancelto{0}{\frac{1}{8}\Big[\sin(2\theta)\Big]_0^{2\pi}}$$

$$= 4\pi + \frac{\pi}{2} = \boxed{\frac{9\pi}{2}}$$

6) $r = 3 + 3\sin\theta$ (cardioid)

θ	r
0	3
$\frac{\pi}{2}$	6
π	3
$\frac{3\pi}{2}$	0
2π	3



$$A = \frac{1}{2}\int_0^{2\pi} (3+3\sin\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} (9 + 18\sin\theta + 9\sin^2\theta)\, d\theta$$

Type 1 even

$$= \frac{9}{2}\int_0^{2\pi} d\theta + 9\int_0^{2\pi} \sin\theta\, d\theta + \frac{9}{2}\int_0^{2\pi} \frac{1-\sin(2\theta)}{2}\, d\theta$$

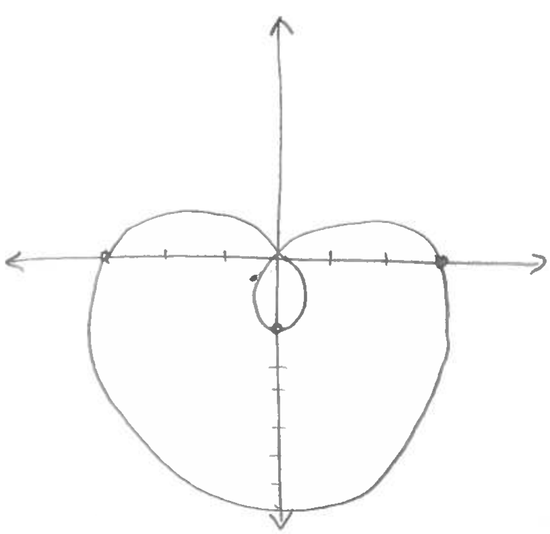
$$= \frac{9}{2}\Big[\theta\Big]_0^{2\pi} + 9\cancelto{0}{\Big[-\cos(\theta)\Big]_0^{2\pi}} + \frac{9}{4}\Big[\theta\Big]_0^{2\pi} - \frac{9}{4}\int_0^{2\pi} \sin(2\theta)\, d\theta$$

$$= 9\pi + \frac{9}{2}\pi - \frac{9}{8}\cancelto{0}{\Big[-\cos(2\theta)\Big]_0^{2\pi}}$$

$$= \boxed{\frac{27}{2}\pi}$$

11) $r = 3 - 4\sin\theta$ (limaçon w/ loop)

θ	r
0	3
$\frac{\pi}{4}$	$3-2\sqrt{2}$
$\frac{\pi}{3}$	$3-2\sqrt{3}$
$\frac{\pi}{6}$	1
$\frac{\pi}{2}$	-1
π	3
$\frac{3\pi}{2}$	7



need to find when $r = 0$ b/c that will be the start/end of the loop

$$r = 0 = 3 - 4\sin\theta$$

$$\sin\theta = \frac{3}{4}$$

$$\theta = \sin^{-1}\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\tfrac{3}{4}\right), \quad \pi - \sin^{-1}\left(\tfrac{3}{4}\right)$$

$$A = \frac{1}{2}\int_{\sin^{-1}(\frac{3}{4})}^{\pi-\sin^{-1}(\frac{3}{4})} (3 - 4\sin\theta)^2 \, d\theta$$

call $a = \sin^{-1}\left(\frac{3}{4}\right)$

$b = \pi - \sin^{-1}\left(\frac{3}{4}\right)$

$$A = \frac{1}{2}\int_a^b 9 - 24\sin\theta + 16\sin^2\theta \, d\theta$$

$$= \frac{1}{2}\int_a^b 9 - 24\sin\theta + 16\left(\frac{1 - \sin 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\int_a^b 17 - 24\sin\theta - 8\sin(2\theta)\, d\theta$$

$$= \frac{17}{2}\Big[\theta\Big]_a^b - 12\Big[-\cos\theta\Big]_a^b - 2\Big[-\cos(2\theta)\Big]_a^b$$

$$= \frac{17}{2}(b-a) + 12\big(\cos(b) - \cos(a)\big) + 2\big(\cos(2b) - \cos(2a)\big)$$

$$= \frac{17}{2}\left(\pi - 2\sin^{-1}\left(\tfrac{3}{4}\right)\right) + 12\left(\cos\left(\pi - \sin^{-1}\left(\tfrac{3}{4}\right)\right) - \cos\left(\sin^{-1}\left(\tfrac{3}{4}\right)\right)\right)$$

double angle formula for cos

$$+ 2\left(\left(1 - 2\sin^2\left(\pi - \sin^{-1}\left(\tfrac{3}{4}\right)\right)\right) - \left(1 - 2\sin^2\left(\sin^{-1}\left(\tfrac{3}{4}\right)\right)\right)\right)$$

addition formulas for cos/sin

$$= \frac{17\pi}{2} - 17\sin^{-1}\left(\tfrac{3}{4}\right) + 12\left(\cos(\pi)\cos\left(\sin^{-1}\left(\tfrac{3}{4}\right)\right) + \overbrace{\cancel{\sin(\pi)\sin\left(\sin^{-1}\left(\tfrac{3}{4}\right)\right)}}^{0}\right)$$

$$- 12\cos\left(\sin^{-1}\left(\tfrac{3}{4}\right)\right) + 2\left(1 - \left[\overbrace{\cancel{\sin(\pi)\cos\left(\sin^{-1}\left(\tfrac{3}{4}\right)\right)}}^{0} - \cos(\pi)\sin\left(\sin^{-1}\left(\tfrac{3}{4}\right)\right)\right]^2\right.$$

$$- 2\left(1 - \left(\frac{3}{4}\right)^2\right)$$

$$= \frac{17}{2}\pi - 17\sin^{-1}\left(\frac{3}{4}\right) + 12(-1)\cos\left(\sin^{-1}\left(\frac{3}{4}\right)\right) - 12\cos\left(\sin^{-1}\left(\frac{3}{4}\right)\right)$$

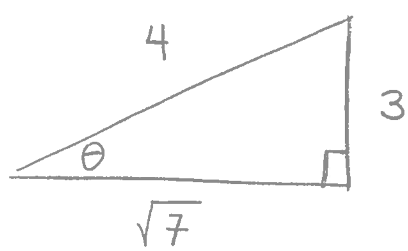
$$+ 2\left(1 - \left(-(-1)\left(\frac{3}{4}\right)\right)^2\right) - 2\left(1 - \frac{9}{16}\right)$$

$$= \frac{17}{2}\pi - 17\sin^{-1}\left(\frac{3}{4}\right) - 24\cos\left(\sin^{-1}\left(\frac{3}{4}\right)\right)$$

$$\cancel{+ 2\left(1 - \frac{9}{16}\right) - 2\left(1 - \frac{9}{16}\right)}$$

note : $\theta = \sin^{-1}\left(\frac{3}{4}\right)$

$\sin\theta = \frac{3}{4}$



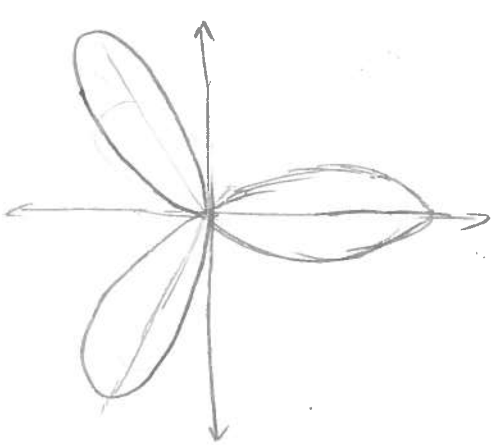
$\Rightarrow \cos\left(\sin^{-1}\left(\frac{3}{4}\right)\right) = \cos(\theta)$

$= \frac{\sqrt{7}}{4}$

$$= \boxed{\frac{17}{2}\pi - 17\sin^{-1}\left(\frac{3}{4}\right) - 6\sqrt{7}}$$

* There may be an algebraic error somewhere

15) $r = 4\cos(3\theta)$



To complete the rose we need $\theta \in [0, \pi]$

$$\Rightarrow A = \frac{1}{2}\int_0^{\pi} \left(4\cos(3\theta)\right)^2 d\theta$$

$$= \frac{1}{2}\int_0^{\pi} 16\cos^2(3\theta)\, d\theta$$

$$= 8\int_0^{\pi}\left(\frac{1+\cos(6\theta)}{2}\right)d\theta$$

$$= 4\int_0^{\pi} d\theta + 4\int_0^{\pi}\cos(6\theta)\,d\theta$$

$$= 4\Big[\theta\Big]_0^{\pi} + 4\left[\frac{\sin(6\theta)}{6}\right]_0^{\pi}$$

$$= 4\pi + \frac{2}{3}\left(\cancel{\sin(12\pi)}^{\,0} - \cancel{\sin(0)}^{\,0}\right)$$

$$= \boxed{4\pi}$$

25) $r = 1 - 2\sin\theta$

slope of tangent line $= \dfrac{f(\theta)\cos(\theta) + f'(\theta)\sin(\theta)}{f(\theta)\sin(\theta) - f'(\theta)\cos(\theta)}$

$$= \frac{(1-2\sin\theta)\cos\theta + (-2\cos\theta)\sin\theta}{(1-2\sin\theta)\sin\theta - (-2\cos\theta)\cos(\theta)}$$

$$= \frac{\cos\theta - 4\sin\theta\cos\theta}{\sin\theta - 2\sin^2\theta + 2\cos^2\theta} = 0$$

$\cancel{\cos\theta} - 4\sin\theta\,\cancel{\cos}\,\theta = 0$

$\theta = \sin^{-1}\left(\frac{1}{4}\right), \pi - \sin^{-1}\left(\frac{1}{4}\right)$

$\cos\theta(1 - 4\sin\theta) = 0$

$\Rightarrow \sin\theta = \frac{1}{4}$ or $\cos(\theta) = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{4}\right) \rightarrow r = 1 - 2\sin\left(\sin^{-1}\left(\frac{1}{4}\right)\right)$

$= 1 - 2\left(\frac{1}{4}\right) = \frac{1}{2}$

$\boxed{\left(\frac{1}{2}, \sin^{-1}\left(\frac{1}{4}\right)\right)}$

$\Rightarrow \quad \theta = \pi - \sin^{-1}\left(\frac{1}{4}\right) \qquad r = 1 - 2\sin\left(\pi - \sin^{-1}\left(\frac{1}{4}\right)\right)$

$r = 1 - 2\left(\cancel{\sin(\pi)\cos}^{\,0}\left(\sin^{-1}\left(\frac{1}{4}\right)\right) - \cos(\pi)\sin\left(\sin^{-1}\left(\frac{1}{4}\right)\right)\right)$

$= 1 - 2\left(-(-1)\left(\frac{1}{4}\right)\right) = 1 - \frac{1}{2} = \frac{1}{2}$

$\boxed{\left(\frac{1}{2}, \pi - \sin^{-1}\left(\frac{1}{4}\right)\right)}$

$\Rightarrow \quad \theta = \frac{\pi}{2} \qquad r = 1 - 2\sin\left(\frac{\pi}{2}\right) = 1 - 2 = -1$

$\boxed{\left(-1, \frac{\pi}{2}\right)}$

$\Rightarrow \quad \theta = \frac{3\pi}{2} \qquad r = 1 - 2\sin\left(\frac{3\pi}{2}\right) = 1 + 2 = 3$

$\boxed{\left(3, \frac{3\pi}{2}\right)}$