Midterm 3 Study Guide Solutions

4.5: 1, 3, 11, 15, 17, 35, 37, 41

1) $f(x) = 4x^3 \qquad x \in [1,3]$

$$\text{Average Value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{3-1}\int_1^3 4x^3\,dx = \frac{1}{2}\left[x^4\right]_1^3$$

$$= \frac{1}{2}\left(3^4 - 1^4\right) = \frac{1}{2}(81-1) = \boxed{40}$$

3) $f(x) = \dfrac{x}{\sqrt{x^2+16}} = x\left(x^2+16\right)^{-1/2} \qquad x \in [0,3]$

$$\text{Average Value} = \frac{1}{3-0}\int_0^3 x\left(x^2+16\right)^{-1/2}dx$$

let $u = x^2+16$

$du = 2x\,dx$

$$= \frac{1}{3}\cdot\frac{1}{2}\int_0^3 2x\left(x^2+16\right)^{-1/2}dx$$

$$= \frac{1}{6}\int_{x=0}^{x=3} u^{-1/2}\,du$$

when $x=0 \quad u=16$

$x=3 \quad u=25$

$$= \frac{1}{6}\int_{u=16}^{u=25} u^{-1/2}\,du = \frac{1}{6}\left[2u^{1/2}\right]_{16}^{25}$$

$$= \frac{1}{3}\left(\sqrt{25}-\sqrt{16}\right) = \frac{1}{3}(5-4) = \boxed{\frac{1}{3}}$$

11) $F(y) = y(1+y^2)^3 \qquad y \in [1,2]$

$$\text{Avg} = \frac{1}{2-1}\int_1^2 y(y^2+1)^3\,dy \qquad \text{let } u = y^2+1, \quad du = 2y\,dy$$

$$= \frac{1}{1}\cdot\frac{1}{2}\int_1^2 2y(y^2+1)^3\,dy = \frac{1}{2}\int_{y=1}^{y=2} u^3\,du$$

when $y=1 \quad u=2$

$y=2 \quad u=5$

$$= \frac{1}{2}\int_{u=2}^{u=5} u^3\,du = \frac{1}{2}\left[\frac{1}{4}u^4\right]_2^5$$

$$= \frac{1}{8}(5^4-2^4) = \frac{1}{8}(625-16) = \frac{1}{8}(609)$$

$$= \boxed{\frac{609}{8}}$$

15) $f(x) = \sqrt{x+1} \qquad x \in [0,3]$

$$f(c) = \frac{1}{3-0}\int_0^3 \sqrt{x+1}\,dx \qquad u = x+1, \quad du = dx \qquad x=0 \Rightarrow u=1, \quad x=3 \Rightarrow u=4$$

$$= \frac{1}{3}\int_{x=0}^{x=3} u^{1/2}\,du = \frac{1}{3}\int_{u=1}^{u=4} u^{1/2}\,du = \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]_1^4$$

$$= \frac{2}{9}\left(4^{3/2}-1^{3/2}\right) = \frac{2}{9}(2^3-1^3) = \boxed{\frac{14}{9}}$$

$$\Rightarrow \quad f(c) = \frac{14}{9} = \sqrt{c+1}$$

$$\Rightarrow \quad c+1 = \frac{14^2}{9^2} = \frac{196}{81}$$

$$c = \frac{196}{81} - \frac{81}{81} = \boxed{\frac{115}{81}}$$

17) $f(x) = 1 - x^2 \qquad x \in [-4, 3]$

$$f(c) = \frac{1}{3-(-4)} \int_{-4}^{3} 1 - x^2 \, dx$$

$$= \frac{1}{7}\left[x - \frac{1}{3}x^3\right]_{-4}^{3} = \frac{1}{7}\left[\left(3 - \frac{1}{3}(3)^3\right) - \left(-4 - \frac{1}{3}(-4)^3\right)\right]$$

$$= \frac{1}{7}\left((3-9) - \left(-4 + \frac{64}{3}\right)\right) = \frac{1}{7}\left(-6 - \frac{52}{3}\right)$$

$$= \frac{1}{7}\left(\frac{-70}{3}\right) = \frac{-10}{3}$$

$$\Rightarrow f(c) = \frac{-10}{3} = 1 - c^2$$

$$\Rightarrow c^2 = 1 + \frac{10}{3} = \frac{13}{3}$$

$$\boxed{c = \pm\sqrt{\frac{13}{3}}}$$

both roots are between $-4, 3$

35) $$\int_{-\pi}^{\pi} \sin(x) + \cos(x) \, dx$$

$\sin(x)$ is odd

$\cos(x)$ is even

$$= \int_{-\pi}^{\pi} \sin(x) + \int_{-\pi}^{\pi} \cos(x) \, dx$$

$$\Rightarrow = 0 + 2\int_{0}^{\pi} \cos(x) \, dx = 2\left[\sin(x)\right]_{0}^{\pi}$$

$$= 2(\sin(\pi) - \sin(0)) = 2(0-0)$$

$$= \boxed{0}$$

37) $$\int_{-\pi/2}^{\pi/2} \frac{\sin(x)}{1+\cos(x)}\,dx$$

note: $$f(-x) = \frac{\sin(-x)}{1+\cos(-x)} = \frac{-\sin(x)}{1+\cos(x)} = -f(x)$$

$$\Rightarrow \quad f(x) = \frac{\sin(x)}{1+\cos(x)} \text{ is odd}$$

$$\Rightarrow \quad \int_{-\pi/2}^{\pi/2} \frac{\sin(x)}{1+\cos(x)}\,dx = \boxed{0}$$

41) $$\int_{-1}^{1} (1+x+x^2+x^3)\,dx$$

note: $1, x^2$ are even

x, x^3 are odd

$$= \underbrace{\int_{-1}^{1} 1+x^2\,dx}_{\text{even}} + \underbrace{\int_{-1}^{1} x+x^3\,dx}_{\text{odd}} = 2\int_{-1}^{1} 1+x^2\,dx + 0$$

$$= 2\left[x+\frac{x^3}{3}\right]_0^1 + 0 = 2\left(1+\frac{1}{3}-0\right) = 2\left(\frac{4}{3}\right)$$

$$= \boxed{\frac{8}{3}}$$

4.6 Concept Review: 3, 4

3) n^4, Parabolic Rule is better than trapezoidal

4) large, trapezoidal overestimates positive concave up functions