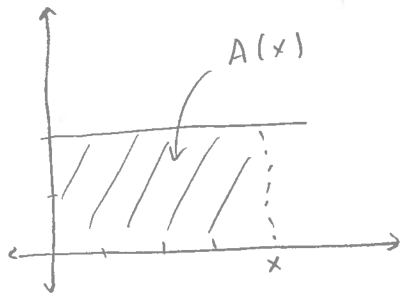
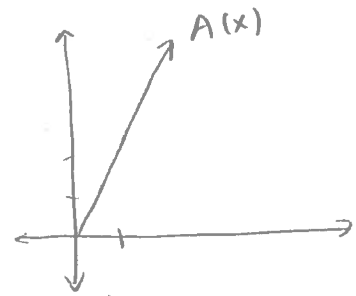
Homework 9 Solutions

4.3 : 1, 3, 6, 9, 12, 17, 23, 24

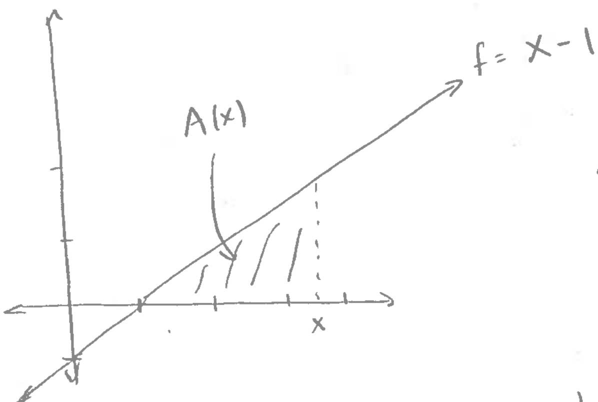
1)



$$A(x) = \int_0^x 2\,dt = \underbrace{2}_{\text{rectangle height}}\underbrace{x}_{\text{base}}$$

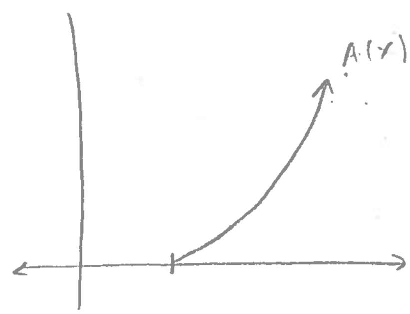


3)

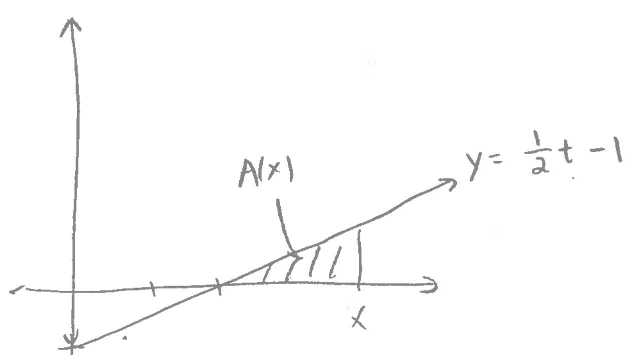


$$A(x) = \int_1^x t - 1\,dt = \frac{1}{2}\underbrace{(x-1)}_{\text{Triangle base}}\underbrace{(x-1)}_{\text{height}}$$

$$= \frac{1}{2}(x-1)^2 \quad , \quad x > 1$$



6)

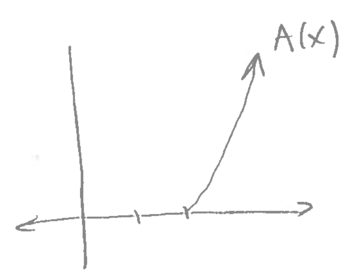


$$A(x) = \int_2^x \left(\frac{1}{2}t - 1\right) dt$$

Triangle

$$= \frac{1}{2}\underbrace{(x-2)}_{\text{base}}\underbrace{\left(\frac{1}{2}x - 1\right)}_{\text{height}} \qquad x > 2$$

$$= \frac{1}{4}x^2 - x + 1$$



9) $\displaystyle\int_1^2 2f(x)\,dx = 2\int_1^2 f(x)\,dx = 2(3) = \boxed{6}$

12) $\displaystyle\int_0^1 \left(2f(s) + g(s)\right) ds = \int_0^1 2f(s)\,ds + \int_0^1 g(s)\,ds$

$\displaystyle = 2\int_0^1 f(s)\,ds + \int_0^1 g(s)\,ds$

$= 2[2] + [-1] = 4 - 1 = \boxed{3}$

17) $\displaystyle G(x) = \int_1^x 2t\,dt \Rightarrow G'(x) = \frac{d}{dx}\int_1^x 2t\,dt$

FFTC $\Rightarrow$ $G'(x) = 2x$

23) $\displaystyle G(x) = \int_1^{x^2} \sin(t)\,dt \Rightarrow G'(x) = \frac{d}{dx}\int_1^{x^2} \sin(t)\,dt$

let $u = x^2$ $\qquad \dfrac{d}{dx} = \dfrac{d}{du}\cdot\dfrac{du}{dx}$ $\qquad \dfrac{du}{dx} = 2x$

$\displaystyle = \frac{d}{dx}\int_1^u \sin(t)\,dt = \frac{du}{dx}\cdot\underbrace{\frac{d}{du}\int_1^u \sin(t)\,dt}_{\text{FFTC}}$

$= 2x\left(\sin(u)\right) = \boxed{2x\sin(x^2)}$

24) $$G(x) = \int_1^{x^2+x} \sqrt{2z + \sin(z)}\, dz$$

$$\Rightarrow \quad G'(x) = \frac{d}{dx} \int_1^{x^2+x} \sqrt{2z + \sin(z)}\, dz$$

let $u = x^2 + x$ $\qquad \frac{du}{dx} = 2x + 1$ $\qquad \frac{d}{dx} = \frac{du}{dx}\frac{d}{du}$

$$= \frac{du}{dx} \frac{d}{du} \int_1^{u} \sqrt{2z + \sin(z)}\, dz$$

$$= (2x+1)\sqrt{2u + \sin(u)} = \boxed{(2x+1)\sqrt{2(x^2+x) + \sin(x^2+x)}}$$

4.4 : 1, 5, 11, 12, 15, 18, 32, 39, 47, 55

1) $$\int_0^2 x^3\, dx = \left[\frac{1}{4}x^4\right]_0^2 = \left(\frac{1}{4}(2)^4 - \frac{1}{4}(0)^4\right) = \boxed{4}$$

5) $$\int_1^4 \frac{1}{w^2}\, dw = \int_1^4 w^{-2}\, dw = \left[-w^{-1}\right]_1^4$$

$$= -\left[\frac{1}{w}\right]_1^4 = -\left(\frac{1}{4} - \frac{1}{1}\right) = -\left(\frac{-3}{4}\right) = \boxed{\frac{3}{4}}$$

11) $$\int_0^{\pi/2} \cos(x)\, dx = \left[\sin(x)\right]_0^{\pi/2} = \left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right) = 1 - 0 = \boxed{1}$$

12) $$\int_{\pi/6}^{\pi/2} 2\sin(t)\, dt = 2\left[-\cos(t)\right]_{\pi/6}^{\pi/2} = -2\left(\cos(\pi/2) - \cos(\pi/6)\right)$$

$$= -2\left(0 - \frac{\sqrt{3}}{2}\right) = \boxed{\sqrt{3}}$$

15) $\int \sqrt{3x+2}\, dx = \int (3x+2)^{1/2}\, dx = \frac{1}{3}\int 3(3x+2)^{1/2}\, dx$

$u = 3x+2$

$du = 3\,dx$

$= \frac{1}{3}\int u^{1/2}\, du$

$= \frac{1}{3}\left(\frac{2}{3}u^{3/2} + C\right) = \frac{2}{9}u^{3/2} + C$

$= \boxed{\frac{2}{9}(3x+2)^{3/2} + C}$

18) $\int \sin(2x-4)\, dx = \frac{1}{2}\int 2\sin(2x-4)\, dx$

$u = 2x-4$

$du = 2\,dx$

$= \frac{1}{2}\int \sin(u)\, du$

$= \frac{1}{2}\left(-\cos(u) + C\right) = \frac{1}{2}\left(-\cos(2x-4) + C\right)$

$= \boxed{-\frac{1}{2}\cos(2x-4) + C}$

32) $\int x^6 \sin(3x^7+9)\sqrt[3]{\cos(3x^7+9)}\, dx$

$u = \cos(3x^7+9)$

$du = -\sin(3x^7+9)(21x^6)\, dx = -21x^6\sin(3x^7+9)$

$= \frac{-1}{21}\int -21x^6 \sin(3x^7+9)\sqrt[3]{\cos(3x^7+9)}\, dx$

$= -\frac{1}{21}\int u^{1/3}\, du = -\frac{1}{21}\left(\frac{3}{4}u^{4/3} + c\right)$

$= \boxed{\frac{-1}{28}\left[\cos(3x^7+9)\right]^{4/3} + C}$

39) $$\int_5^8 \sqrt{3x+1}\,dx$$

$u = 3x+1$

$du = 3\,dx$

$$= \frac{1}{3}\int_5^8 3\sqrt{3x+1}\,dx = \frac{1}{3}\int_{x=5}^{x=8} u^{1/2}\,du$$

when $x = 5 \rightarrow u = 16$

$x = 8 \rightarrow u = 25$

$$= \frac{1}{3}\int_{u=16}^{u=25} u^{1/2}\,du = \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]_{16}^{25}$$

$$= \frac{2}{9}\left[25^{3/2} - 16^{3/2}\right] = \frac{2}{9}\left(5^3 - 4^3\right)$$

$$= \frac{2}{9}(125 - 64) = \frac{2 \cdot 61}{9} = \boxed{\frac{122}{9}}$$

47) $$\int_0^{\pi/6} \sin^3(\theta)\cos(\theta)\,d\theta$$

$u = \sin(\theta)$

$du = \cos(\theta)\,d\theta$

when $\theta = 0$ $u = 0$

$\theta = \frac{\pi}{6}$ $u = \frac{1}{2}$

$$= \int_{\theta=0}^{\theta=\pi/6} u^3\,du = \int_{u=0}^{u=1/2} u^3\,du = \left[\frac{1}{4}u^4\right]_0^{1/2} = \frac{1}{4}\left(\left(\frac{1}{2}\right)^4 - (0)^4\right)$$

$$= \frac{1}{4}\cdot\frac{1}{2^4} = \frac{1}{2^6} = \boxed{\frac{1}{64}}$$

55) $$\int_0^{\pi/2} \sin(x)\sin(\cos(x))\,dx$$

$u = \cos(x)$

$du = -\sin(x)$

$$= -\int_0^{\pi/2} -\sin(x)\sin(\cos(x))\,dx$$

$$= -\int_{x=0}^{x=\pi/2} \sin(u)\,du$$

when $x = 0$ $\quad u = \cos(0) = 1$

$x = \pi/2$ $\quad u = \cos(\pi/2) = 0$

$$= -\int_{u=1}^{u=0} \sin(u)\,du = -\Big[-\cos(u)\Big]_1^0 = \Big[\cos(u)\Big]_1^0$$

$$= \big(\cos(0) - \cos(1)\big) = \boxed{1 - \cos(1)}$$