Homework 8 Solutions

4.1: 1, 2, 6, 10, 11, 13

1) $$\sum_{k=1}^{6} (k-1) = \sum_{k=1}^{6} k - \sum_{k=1}^{6} 1 = \frac{(6+1)(6)}{2} - 6\cdot 1$$

$$= 7\cdot 3 - 6 = 21 - 6 = \boxed{15}$$

2) $$\sum_{i=1}^{6} i^2 = \frac{6(6+1)(2\cdot 6+1)}{6} = 7(13) = \boxed{91}$$

6) $$\sum_{k=3}^{7} \frac{(-1)^k 2^k}{(k+1)} = \underbrace{\frac{(-1)^3 2^3}{3+1}}_{k=3} + \underbrace{\frac{(-1)^4 2^4}{4+1}}_{k=4} + \underbrace{\frac{(-1)^5 2^5}{5+1}}_{k=5} + \underbrace{\frac{(-1)^6 2^6}{6+1}}_{k=6}$$

$$+ \underbrace{\frac{(-1)^7 2^7}{7+1}}_{k=7} = \frac{-8}{4} + \frac{16}{5} - \frac{32}{6} + \frac{64}{7} - \frac{128}{8}$$

$$= -2 + \frac{16}{5} - \frac{16}{3} + \frac{64}{7} - 16 = \boxed{\frac{-1154}{105}}$$

$$\approx -10.99$$

10) $$\underbrace{2+4+6+8+\cdots 50}_{25 \text{ terms}} \Rightarrow \boxed{\sum_{i=1}^{25} 2i}$$

when $i=1 \Rightarrow 2$

$i=2 \longrightarrow 4 \Rightarrow$ pattern is $2i$

$i=3 \longrightarrow 6$

11) $\underbrace{1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{100}}_{100 \text{ terms}} \Rightarrow \sum_{i=1}^{100} \Rightarrow \boxed{\sum_{i=1}^{100} \frac{1}{i} = \sum_{i=1}^{100} i^{-1}}$

when $i=1 \to 1$

$i=2 \to \frac{1}{2} \Rightarrow$ pattern is $\frac{1}{i}$

$i=3 \to \frac{1}{3}$

13) $\underbrace{a_1 + a_3 + a_5 + a_7 + \cdots + a_{99}}_{50 \text{ terms}} \Rightarrow \sum_{i=1}^{50} \Rightarrow \boxed{\sum_{i=1}^{50} a_{2i-1}}$

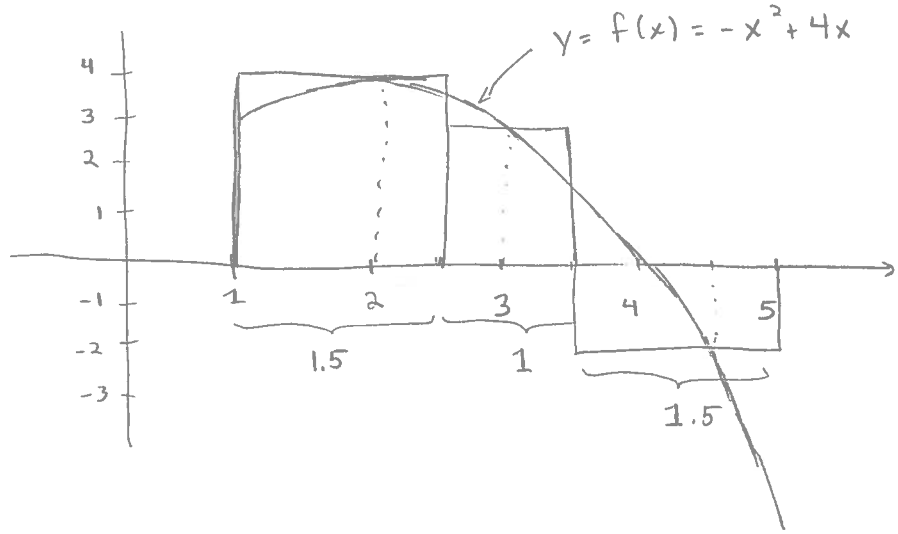
when $i=1 \to a_1$

$i=2 \to a_3 \Rightarrow$ index goes like $2i-1$

$i=3 \to a_5 \Rightarrow$ pattern is a_{2i-1}

4.2 : 1, 3, 7, 11, 12, 17, 27

1)



$f(2) = -(2)^2 + 4(2) = 4$

$f(3) = -(3)^2 + 4(3) = 3$

$f(4.5) = -(4.5)^2 + 4(4.5)$

$= -2.25$

Riemann Sum $= f(2)\cdot 1.5 + f(3)\cdot 1 + f(4.5)(1.5) = 4(1.5) + 3(1) + (-2.25)(1.5)$

$= \boxed{5.625}$

3) $f(x) = x - 1$ $\quad P: 3 < 3.75 < 4.25 < 5.5 < 6 < 7$

$\bar{x}_1 = 3,\ \bar{x}_2 = 4,\ \bar{x}_3 = 4.75,\ \bar{x}_4 = 6,\ \bar{x}_5 = 6.5$

from P we get that

$\Delta x_1 = 3.75 - 3 = 0.75$

$\Delta x_2 = 4.25 - 3.75 = 0.5$

$\Delta x_3 = 5.5 - 4.25 = 0.75$

$\Delta x_4 = 6 - 5.5 = 0.5$

$\Delta x_5 = 7 - 6 = 1$

$$\Rightarrow R = f(3)(0.75) + f(4)(0.5) + f(4.75)(0.75) + f(6)(0.5) + f(6.5)$$

$$= 2\left(\tfrac{3}{4}\right) + 3\left(\tfrac{1}{2}\right) + 3.75\left(\tfrac{3}{4}\right) + 5\left(\tfrac{1}{2}\right) + 5.5$$

$$= 1.5 + 1.5 + \frac{45}{16} + 2.5 + 5.5 = \boxed{15.6875}$$

7) $\lim\limits_{\|P\| \to 0} \sum\limits_{i=1}^{n} (\bar{x}_i)^3 \Delta x_i$; $a = 1,\ b = 3$

$f(\bar{x}_i) = \bar{x}_i^{\,3} \Rightarrow f(x) = x^3$

$$\Rightarrow \boxed{\int_1^3 x^3\,dx}$$

11) $$\int_0^2 (x+1)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\Delta x = \text{width} = \frac{\text{total width}}{\#\text{rectangles}} = \frac{2}{n}$$

$$x_i = a + i\Delta x = 0 + \frac{2i}{n} = \frac{2i}{n}$$

$$f(x_i) = x_i + 1 = \frac{2i}{n} + 1$$

$$\Rightarrow \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n} + 1\right)\left(\frac{2}{n}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4i}{n^2} + \frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{4}{n^2}\underbrace{\sum_{i=1}^{n} i}_{\frac{(n+1)n}{2}} + \frac{2}{n}\underbrace{\sum_{i=1}^{n} 1}_{n} = \lim_{n\to\infty} \frac{4(n^2+n)}{2n^2} + \frac{2n}{n}$$

$$= \lim_{n\to\infty} \frac{4n^2}{2n^2} + \frac{4n}{2n^2} + \frac{2n}{n}$$

$$= \lim_{n\to\infty} 2 + \cancel{\frac{2}{n}}^{\to 0} + 2 = \boxed{4}$$

12) $$\int_0^2 (x^2+1)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\Delta x = \frac{2}{n}$$

$$x_i = a + i\Delta x = 0 + \frac{2i}{n} = \frac{2i}{n}$$

$$f(x_i) = x_i^2 + 1 = \left(\frac{2i}{n}\right)^2 + 1 = \frac{4i^2}{n^2} + 1$$

$$\Rightarrow \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} + 1 \right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i^2}{n^3} + \frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i^2}{n^3} + \sum_{i=1}^{n} \frac{2}{n} = \lim_{n\to\infty} \frac{8}{n^3} \underbrace{\sum_{i=1}^{n} i^2}_{\frac{n(n+1)(2n+1)}{6}} + \frac{2}{n} \underbrace{\sum_{i=1}^{n} 1}_{n}$$

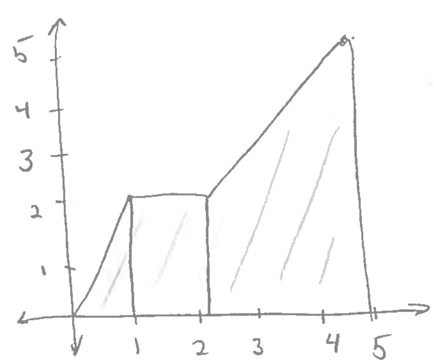
$$= \lim_{n\to\infty} \frac{8}{n^3} \cdot \frac{(2n^3 + 3n^2 + n)}{6} + \frac{2n}{n}$$

$$= \lim_{n\to\infty} \frac{16n^3}{6n^3} + \frac{24n^2}{6n^3} + \frac{8n}{6n^3} + \frac{2n}{n}$$

$$= \lim_{n\to\infty} \frac{16}{6} + \cancel{\frac{4}{n}}^{\,0} + \cancel{\frac{8}{6n^2}}^{\,0} + 2$$

$$= \frac{16}{6} + 2 = \frac{28}{6} = \boxed{\frac{14}{3}}$$

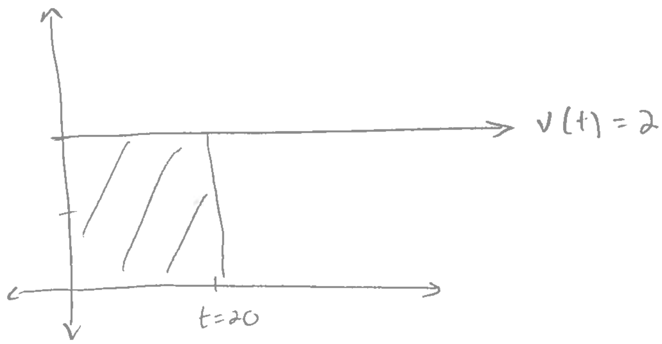
17)



$$= \underset{\text{Triangle}}{\frac{1}{2}(1)(2)} + \underset{\text{Rectangle}}{1(2)} + \underset{\text{Trapezoid}}{\frac{(5+2)}{2}(3)}$$

$$= 1 + 2 + \frac{21}{2} = \boxed{\frac{27}{2}}$$

27)



$d(20) = \int_0^{20} v(t)\,dt = \int_0^{20} 2\,dt$ = rectangle of base = 20, height = 2 $= \boxed{40}$

$d(40)$ = rectangle base = 40, height = 2 $= \boxed{80}$

$d(60) = 60 \times 2 = \boxed{120}$

$d(80) = 80 \times 2 = \boxed{160}$

$d(100) = 100 \times 2 = \boxed{200}$

$d(120) = 120 \times 2 = \boxed{240}$