HW 7 Solutions

3.5 : 1, 4, 9, 16, 25, 29

3.6 : 1, 5, 19

3.8 : 1, 5, 8, 14, 27, 30

Chapter 3 Section 5 :

1) $f(x) = x^3 - 3x + 5$

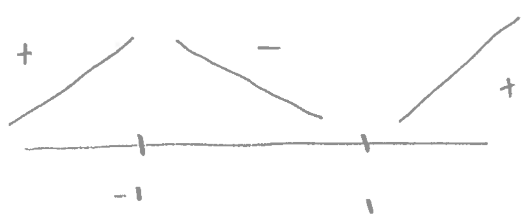
x-intercepts → no integer, 1 between $x = -2$, $x = -3$

y-intercept $f(0) = 5$

$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x+1)(x-1)$

roots a $x = 1$, $x = -1$ both single roots

f'



$f'(0) = 3(1)(-1) < 0$

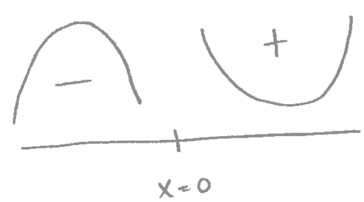
$\Rightarrow$ $x = -1$ local max

$x = 1$ local min

increasing on $(-\infty, -1] \cup [1, \infty)$

decreasing on $[-1, 1]$

f''



$f''(x) = 6x = 0$

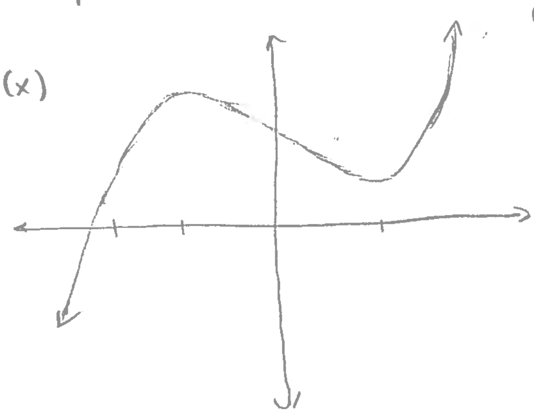
1 root at $x = 0$

inflection point at $x = 0$

concave down on $(-\infty, 0]$

Concave up on $[0, \infty)$

f(x)



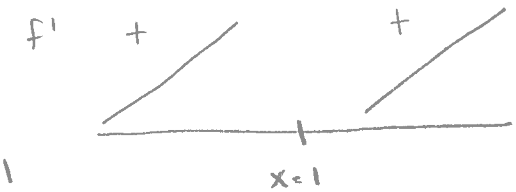
4) $f(x) = (x-1)^3$

x-intercepts $x = 1$ (triple root)

y-intercept $f(0) = (-1)^3 = -1$

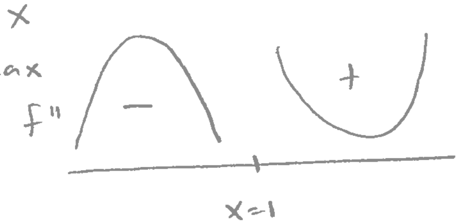
$f'(x) = 3(x-1)^2(1) = 3(x-1)^2 = 0$

double root at $x = 1$



increasing for all x

no local min/max



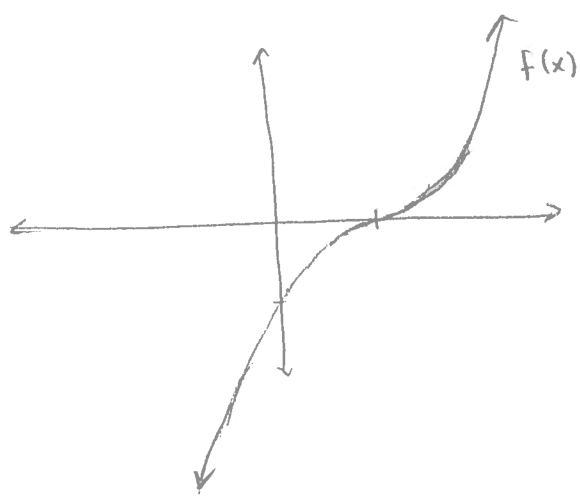
$f''(x) = 6(x-1)(1) = 6(x-1) = 0$

single root at $x = 1$

concave down for $(-\infty, 1]$

concave up for $[1, \infty)$

inflection point at $x = 1$



9) $g(x) = \dfrac{x}{x+1}$

x-intercepts: $g(x) = 0 = \dfrac{x}{x+1} \Rightarrow x = 0$

y-intercept $g(0) = 0$

vertical asymptote at $x = -1$

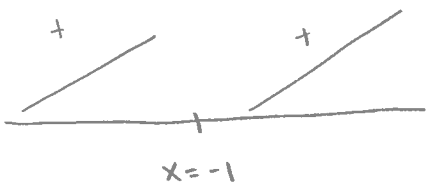
$\lim_{x \to \infty} g(x) = 1$, $\lim_{x \to -\infty} g(x) = 1$ $\Rightarrow$ horizontal asymptote at $y = 1$

$$g'(x) = \frac{\frac{d}{dx}[x](x+1) - x\frac{d}{dx}[x+1]}{(x+1)^2}$$

$$= \frac{(x+1) - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2} = 0 \quad \text{never}$$

$x = -1$ is a $g'(x)$ DNE critical point (division by 0)

g'



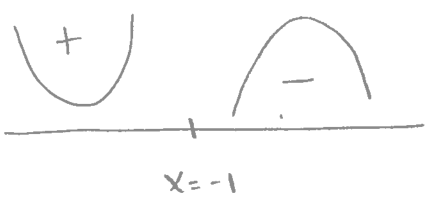
g increasing for all x

$$g''(x) = \frac{d}{dx}\left[(x+1)^{-2}\right] = -2(x+1)^{-3}(1)$$

$$= \frac{-2}{(x+1)^3} = 0 \quad \text{never}$$

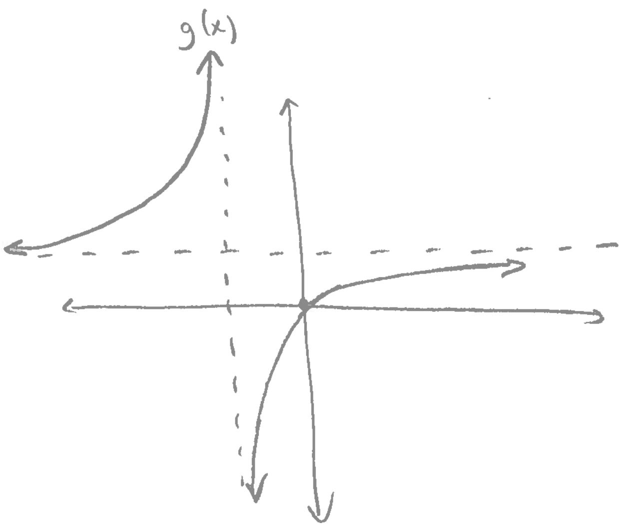
$x = -1$ causes division by 0

g''



$$g''(0) = \frac{-2}{1^3} < 0$$

$$g''(-2) = \frac{-2}{-1^3} > 0$$



16) $w(z) = \dfrac{z^2+1}{z}$

x-intercepts : $\dfrac{z^2+1}{z} = 0$ $\quad z^2+1=0$ $\quad$ no real roots

y-intercept : $w(0) = \dfrac{1}{0}$ $\quad$ undefined

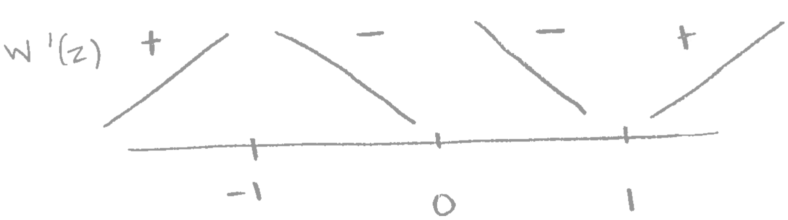
vertical asymptote at $z=0$

$$w'(z) = \frac{\frac{d}{dz}[z^2+1]\,z - \frac{d}{dz}[z]\,(z^2+1)}{z^2}$$

$$= \frac{(2z)(z) - (1)(z^2+1)}{z^2} = \frac{z^2-1}{z^2} = 0$$

$\Rightarrow$ $w'(z) = 0 \iff z^2-1=0 \Rightarrow z=1,\ z=-1$ both single roots

$w'(z)$ DNE for $z=0$ (division by 0)



$w'(-2) = \dfrac{(-2)^2-1}{(-2)^2} = \dfrac{3}{4} > 0$

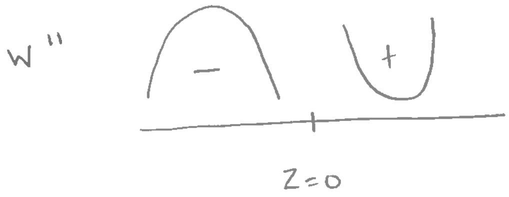
$w'(2) = \dfrac{(2)^2-1}{2^2} = \dfrac{3}{4} > 0$

$\Rightarrow$ local min at $z=1$ $\quad$ increasing for $(-\infty,-1] \cup [1,\infty)$

local max at $z=-1$ $\quad$ decreasing for $[-1,1]$

$$w''(z) = \frac{\frac{d}{dz}[z^2-1]\,z^2 - \frac{d}{dz}[z^2]\,(z^2-1)}{z^4} = \frac{(2z)(z^2) - (2z)(z^2-1)}{z^4}$$

$$= \frac{2z^3 - (2z^3-2z)}{z^4} = \frac{2z}{z^4}$$ undefined at $z=0$

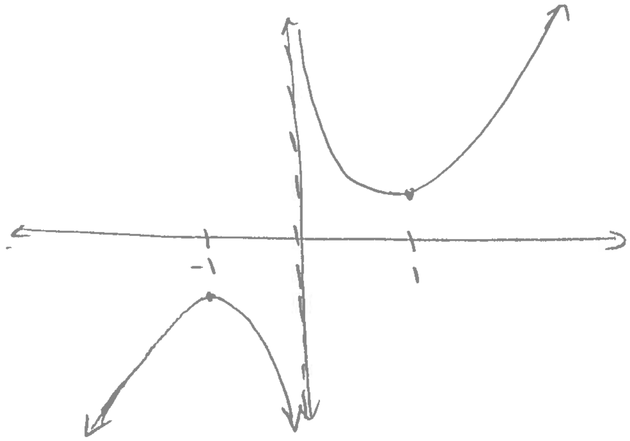


$$w''(1) = \frac{2(1)}{(1)^4} = 2 > 0$$

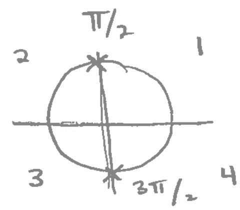
$$w''(-1) = \frac{2(-1)}{(-1)^4} = -2 < 0$$

concave up for $[0,\infty)$

concave down for $(-\infty, 0]$



25) $h(t) = \cos^2(t)$



x-intercepts $\Rightarrow$ $\cos^2(t) = 0$

$\cos(t) = 0$ $\qquad t = \frac{\pi}{2} \pm n\pi$

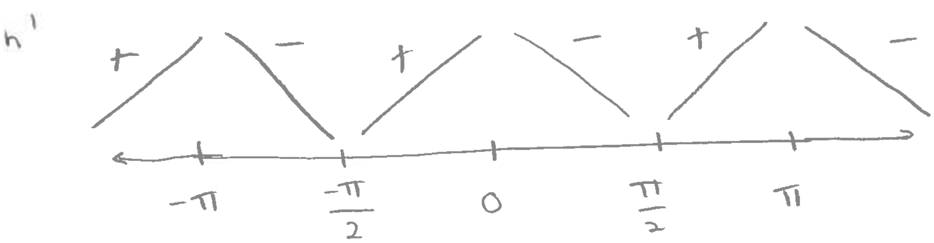
y-intercept $\quad h(0) = \cos^2(0) = 1^2 = 1$

$$h'(t) = 2\cos(t)\,(-\sin(t))$$

$$= -2\sin(t)\cos(t)$$

is negative when sin, cos have same sign (Quadrants 1 + 3)

is positive when sin, cos have different sign (Quadrants 2, 4)



local max at $-\pi, 0, \pi, \ldots$

local min at $-\frac{\pi}{2}, \frac{\pi}{2}, \ldots$

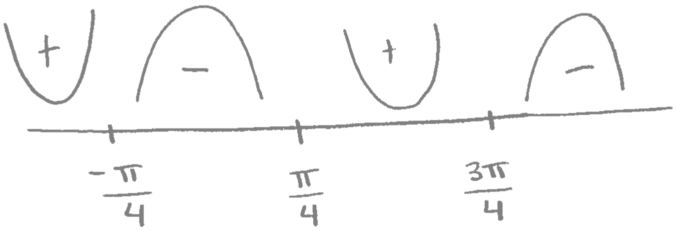
$$h''(t) = -2\left(\frac{d}{dt}[\sin(t)]\cos(t) + \frac{d}{dt}[\cos(t)]\sin(t)\right)$$

$$= -2\left(\cos^2(t) - \sin^2(t)\right)$$

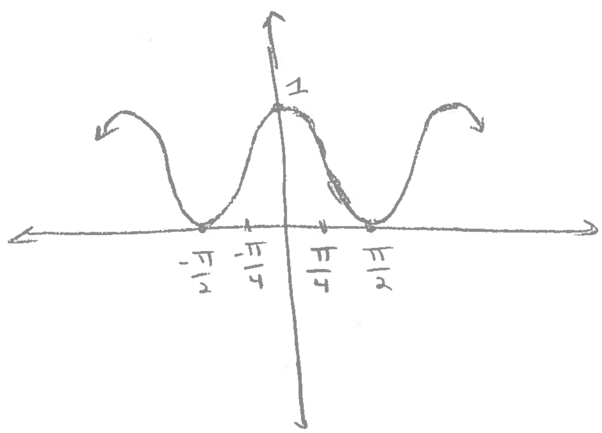
$$= -2\cos(2t) = 0$$

$$\cos(2t) = 0$$

$$t = \frac{\pi}{4} \pm \frac{\pi}{2}$$



$$h''(0) = -2\cos(0) = -2 < 0$$



29) f continuous everywhere

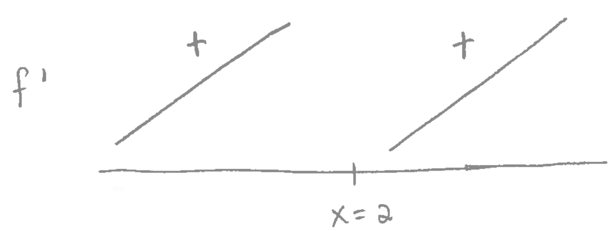
$f(2) = -3, \quad f(6) = 1$

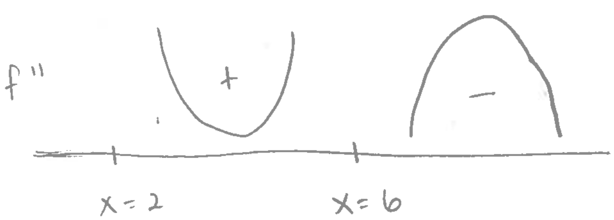
$f'(2) = 0, \quad f'(x) > 0$ for $x \neq 2$ $\Rightarrow$ always increasing

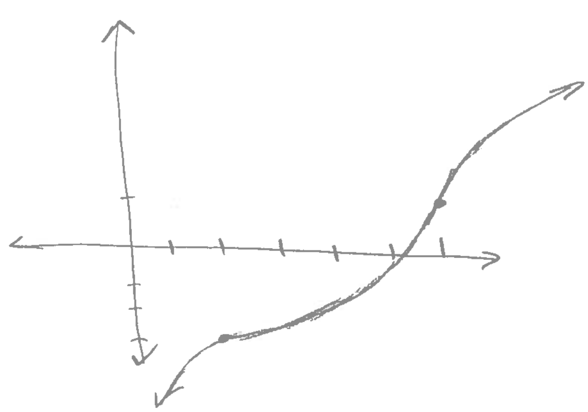
$f'(6) = 3, \quad f''(6) = 0 \qquad f''(x) > 0$ for $2 < x < 6$

$f''(x) < 0$ for $x > 6$

$\Rightarrow x = 6$ is an inflection point







Chapter 3 Section 6:

1) $f(x) = |x| \qquad I = [1,2]$

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases} \Rightarrow \text{for } x \in I \quad f(x) = |x| = x$$

$$\left.\begin{array}{l} f \text{ is continuous on } [1,2] \\ f \text{ is differentiable on } (1,2) \end{array}\right\} \text{polynomial}$$

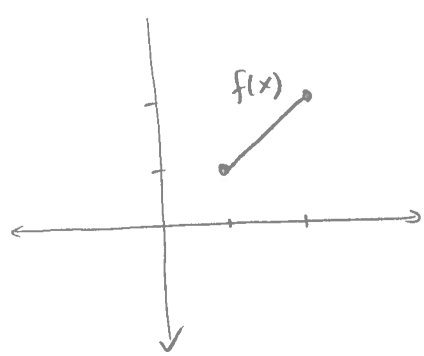
$\Rightarrow$ MVT holds

$$\left.\begin{array}{l} f(1) = |1| = 1 \\ f(2) = |2| = 2 \end{array}\right. \Rightarrow \begin{array}{l} (1,1) \\ (2,2) \end{array} \text{ are the endpoints}$$

Slope of the secant line connecting the endpoints

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{2-1} = \frac{1}{1} = 1$$

$f'(x) = \frac{d}{dx}[x] = 1 \Rightarrow$ any $x \in I$ satisfies the MVT



5) $H(s) = s^2 + 3s - 1 \qquad I = [-3,1]$

$H(s)$ is a polynomial $\Rightarrow$ continuous on $[-3,1]$, differentiable on $(-3,1)$ $\Rightarrow$ MVT holds

$H(-3) = (-3)^2 + 3(-3) - 1 = 9 - 9 - 1 = -1$

$H(1) = (1)^2 + 3(1) - 1 = 3$

$\Rightarrow$ $(-3,-1)$, $(1,3)$ are enpoints

Secant line slope $= \dfrac{3-(-3)}{1-(-1)} = \dfrac{6}{2} = 3$

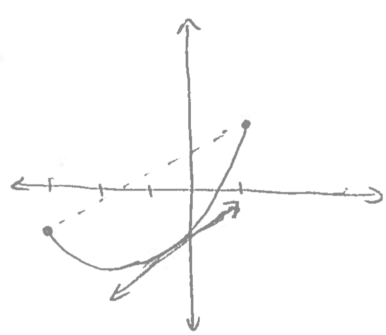
tangent line slope $= h'(s) = 2s + 3$

$\Rightarrow \quad 2s + 3 = 3$

$2s = 0$

$s = 0$

$\Rightarrow$ the point $c = 0$ satisfies the MVT



19) $f(x) = x + \dfrac{1}{x} \qquad I = [1,2]$

$f(x) = \dfrac{x^2+1}{x}$ is continuous and differentiable except at $x=0$

as $x=0 \notin [1,2]$ MVT holds

$f(1) = \dfrac{1^2+1}{1} = 2$

$f(2) = \dfrac{2^2+1}{2} = \dfrac{5}{2}$

$\Rightarrow$ $(1,2)$, $(2, \frac{5}{2})$ are endpoints

Secant line slope $= \dfrac{2 - \frac{5}{2}}{1 - 2} = \dfrac{-\frac{1}{2}}{-1} = \dfrac{1}{2}$

$f'(x) = 1 + (-1)x^{-2}$

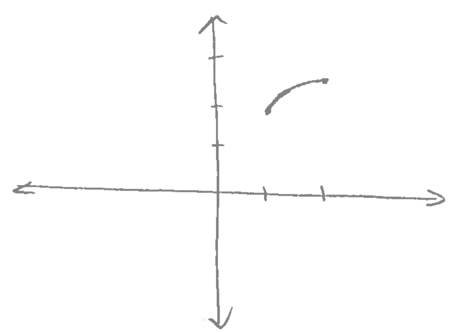
$= 1 - \dfrac{1}{x^2} = \dfrac{x^2 - 1}{x^2} = \dfrac{1}{2}$

$\Rightarrow 2(x^2 - 1) = x^2$

$2x^2 - 2 = x^2$ only $c = \sqrt{2} \in [1,2]$

$x^2 - 2 = 0$

$x = \pm\sqrt{2}$



Chapter 3 Section 8:

1) $f(x) = 5$

$\Rightarrow$ anti derivative is $\boxed{5x + C}$ (Power Rule)

5) $f(x) = x^{5/4}$

$\Rightarrow$ antiderivative is $a x^{9/4} + C$

where $\dfrac{9}{4} a = 1 \Rightarrow a = \dfrac{4}{9}$

$\Rightarrow \boxed{F(x) = \dfrac{4}{9} x^{9/4} + C}$ (Power Rule)

8) $f(x) = 7x^{-3/4}$

$$\Rightarrow \quad F(x) = ax^{1/4} + C$$

where $\frac{1}{4}a = 7 \quad \Rightarrow \quad a = 28$

$$\boxed{F(x) = 28x^{1/4} + C}$$

14) $f(x) = x^2(x^3 + 5x^2 - 3x + \sqrt{3})$

$$= x^5 + 5x^4 - 3x^3 + \sqrt{3}\,x^2$$

$$\Rightarrow \quad F(x) = ax^6 + bx^5 - cx^4 + dx^3 + C$$

where

$$\begin{aligned} 6a &= 1 & & a = \tfrac{1}{6} \\ 5b &= 5 & & b = 1 \\ -4c &= -3 & \Rightarrow \quad & c = \tfrac{3}{4} \\ 3d &= \sqrt{3} & & d = \tfrac{\sqrt{3}}{3} \end{aligned}$$

$$\Rightarrow \quad \boxed{F(x) = \frac{1}{6}x^6 + x^5 - \frac{3}{4}x^4 + \frac{\sqrt{3}}{3}x^3 + C}$$

27) $\int (\sqrt{2}x + 1)^3 \sqrt{2}\, dx$

note: if $u = \sqrt{2}x + 1$

$du = \sqrt{2}\, dx$

$$\Rightarrow \quad \int u^3\, du = \frac{1}{4}u^4 + C$$

$$\boxed{= \frac{1}{4}(\sqrt{2}x + 1)^4 + C}$$

30) $\int (5x^2+1)\sqrt{5x^3+3x-2}\; dx$

$$= \int (5x^2+1)(5x^3+3x-2)^{1/2}\, dx$$

note if $u = 5x^3+3x-2$

$du = (15x^2+3)\,dx$ which is 3 times $5x^2+1$

$$\Rightarrow \int \tfrac{1}{3}\cdot 3\,(5x^2+1)(5x^3+3x-2)^{1/2}\, dx$$

$$= \frac{1}{3}\int \underbrace{(5x^3+3x-2)^{1/2}}_{u^{1/2}}\underbrace{(15x^2+3)\, dx}_{du}$$

$$= \frac{1}{3}\int u^{1/2}\, du = \frac{1}{3}\left[\frac{2}{3}u^{\frac{3}{2}} + C\right]$$

$$= \frac{2}{9}u^{\frac{3}{2}} + C$$

$$= \boxed{\frac{2}{9}(5x^3+3x-2)^{3/2} + C}$$