Assignment 5 Solutions

2.9 : 1, 4, 37, 44

3.1 : 5, 14, 17, 21

3.2 : 3, 9, 10, 13, 18, 21

Chapter 2 Section 9 :

1) $y = x^2 + x - 3$

$$dy = 2x\,dx + dx$$

$$dy = (2x+1)\,dx$$

4) $y = (3x^2 + x + 1)^{-2}$ (chain rule)

$$dy = -2(3x^2+x+1)^{-3}(6x\,dx + dx)$$

$$dy = -2(6x+1)(3x^2+x+1)^{-3}\,dx$$

37) $f(x) = x^2$ at $a = 2$, interval $= [0,3]$

for a linear approximation we need a slope and a point

Slope : $f'(a)$

$f'(x) = 2x \Rightarrow f'(a) = f'(2) = 2(2) = 4$ slope $= 4$

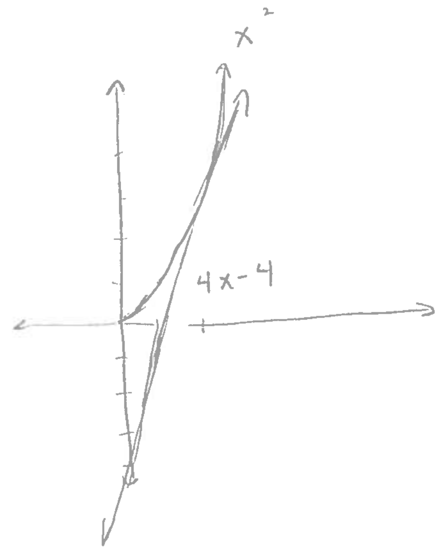
point : $(a, f(a)) = (2, 2^2) = (2,4)$

point - slope form

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 4(x-2)$$

$$\boxed{y = 4x - 4}$$



44) $G(x) = x + \sin(2x)$, $a = \frac{\pi}{2}$, interval $= [0, \pi]$

Slope $= G'(a)$

$G'(x) = 1 + 2\cos(2x)$

$G'(a) = 1 + 2\cos\left(2 \cdot \frac{\pi}{2}\right) = 1 + 2\cos(\pi) = 1 + 2(-1) = -1$

point $= \left(\frac{\pi}{2}, G\left(\frac{\pi}{2}\right)\right)$

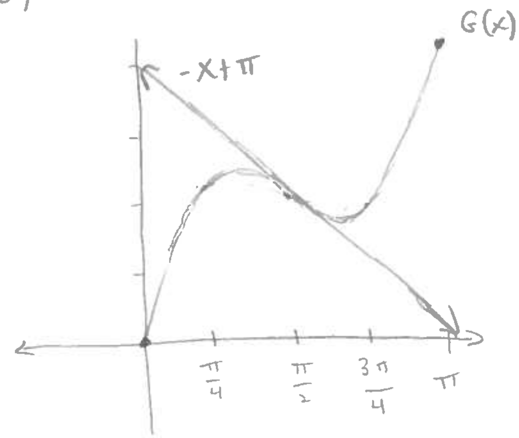
$= \left(\frac{\pi}{2}, \frac{\pi}{2} + \sin\left(2 \cdot \frac{\pi}{2}\right)\right) = \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$y - \frac{\pi}{2} = -1\left(x - \frac{\pi}{2}\right)$

$y = -x + \pi$

G(x)

x	y
0	0
$\frac{\pi}{4}$	$1 + \frac{\pi}{4}$
$\frac{\pi}{2}$	$\frac{\pi}{2}$
$\frac{3\pi}{4}$	$\frac{3\pi}{4} - 1$
π	π



Section 3.1

5) $f(x) = x^2 + 4x + 4$ $\quad I = [-4, 0]$

Critical Points: endpoints: $x = -4$, $x = 0$

$f'(x) = 0$:

$f'(x) = 2x + 4 = 0$

$x = -2$

$f'(x)$ DNE, no points

$f(-4) = (-4)^2 + 4(-4) + 4$

$= 16 - 16 + 4$

$= 4$

$f(-2) = (-2)^2 + 4(-2) + 4$

$= 4 - 8 + 4$

$= 0$

$f(0) = 4$

maximum value $= 4$ occurs at $x = 0, -4$

minimum value $= 0$ occurs at $x = -2$

14) $f(x) = x^5 - \frac{25}{3}x^3 + 20x - 1$ $\qquad I = [-3, 2]$

critical points: endpoints: $x = -3,\ x = 2$

$f'(x) = 0$: $f'(x) = 5x^4 - 25x^2 + 20$

$$= 5(x^4 - 5x^2 + 4)$$

$$= 5(x^2 - 4)(x^2 - 1) = 0$$

$x = 2,\ x = -2,\ x = 1,\ x = -1$

$f'(x)$ DNE : no points

$f(-2) = -\frac{19}{3}$

$f(-3) = -79$

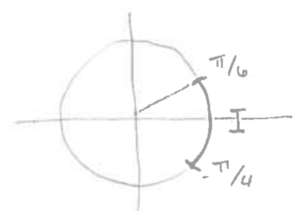
$f(-1) = -\frac{41}{3}$ $\Rightarrow$ maximum value $= \frac{35}{3}$ at $x = 1$

minimum value $= -79$ at $x = -3$

$f(1) = \frac{35}{3}$

$f(2) = \frac{13}{3}$

17) $r(\theta) = \sin\theta$ $\qquad I = \left[-\frac{\pi}{4}, \frac{\pi}{6}\right]$



critical points: endpoints: $x = -\frac{\pi}{4}, \frac{\pi}{6}$

$\cos\theta \neq 0$ in I

$r'(\theta) = 0$: $r'(\theta) = \cos(\theta) = 0$

$\Rightarrow \theta = \pm\frac{\pi}{2}$ but these points are not in I

$\Rightarrow$ no points

$r'(\theta)$ undefined : no points

$r\left(-\frac{\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) = \frac{-\sqrt{2}}{2}$ $\qquad r\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

max value $= \frac{1}{2}$ at $x = \frac{\pi}{6}$ $\qquad$ min value $= -\frac{\sqrt{2}}{2}$ at $x = -\frac{\pi}{4}$

21) $g(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$ $\qquad I = [-1, 27]$

critical points: endpoints: $x = -1, \quad x = 27$

$g'(x) = 0 \qquad g'(x) = \frac{1}{3} x^{-2/3}$

$$= \frac{1}{3\sqrt[3]{x^2}} = 0$$

nowhere in I

$g'(x)$ undefined if $x = 0$ (divide by 0)

$g(-1) = (-1)^{1/3} = -1$

maximum value $= 3$ at $x = 27$

$g(0) = (0)^{1/3} = 0 \quad \Rightarrow$

minimum value $= -1$ at $x = -1$

$g(27) = (27)^{1/3} = 3$

Section 3.2:

3) $h(t) = t^2 + 2t - 3$ $\qquad h$ is a quadratic polynomial

$\Rightarrow$ continuous and differentiable

$h'(t) = 2t + 2 = 0$

for $t < -1$ $\quad h'(t) < 0 \Rightarrow$ decreasing

$t = -1$

for $t > -1$ $\quad h'(t) > 0 \Rightarrow$ increasing

increasing for $t \in [-1, \infty)$

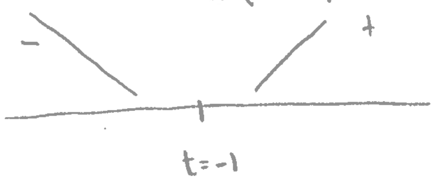
decreasing for $t \in (-\infty, -1]$

Chapter 3 Section 2:

3) $h(t) = t^2 + 2t - 3$

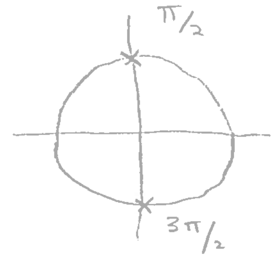
$\Rightarrow h'(t) = 2t + 2$

$= 2(t+1)$ single root at $t = -1$

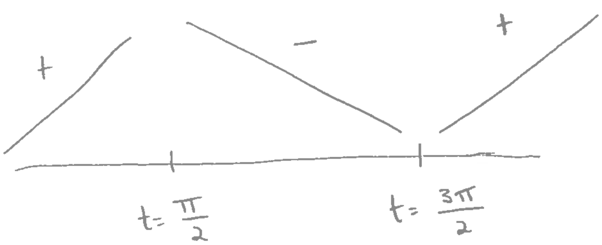


$\boxed{\begin{array}{ll} \text{decreasing on} & (-\infty, -1] \\ \text{increasing on} & [-1, \infty) \end{array}}$

9) $H(t) = \sin(t)$ $\quad 0 \le t \le 2\pi$



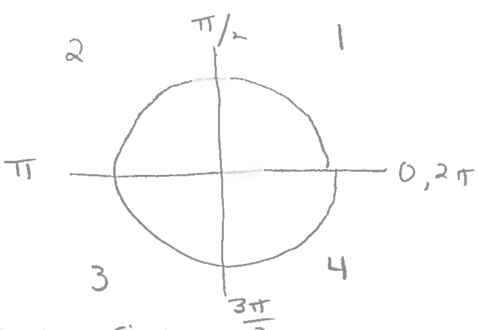
$H'(t) = \cos(t)$



increasing on $[0, \frac{\pi}{2}] \cup [\frac{3\pi}{2}, 2\pi]$

decreasing on $[\frac{\pi}{2}, \frac{3\pi}{2}]$

10) $R(\theta) = \cos^2\theta$, $\quad 0 \le \theta \le 2\pi$



$R'(\theta) = 2\cos\theta(-\sin\theta)$

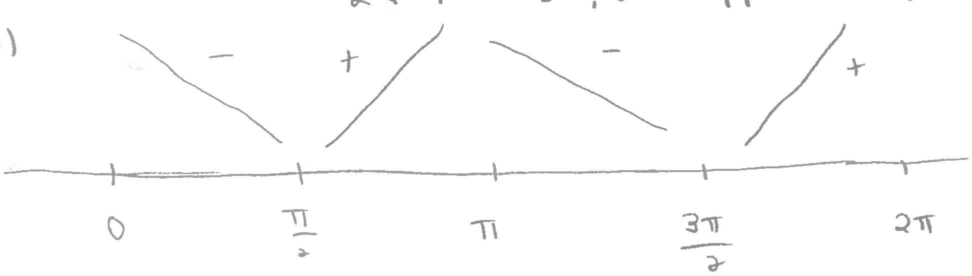
$= -2\cos\theta \sin\theta$

in Quadrants 1 + 3 sin, cos same sign

2 + 4 sin, cos opposite sign

$R'(\theta)$

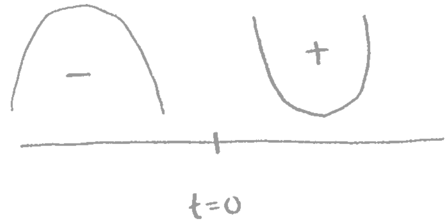
$\Rightarrow$



increasing on $[\pi/2, \pi] \cup [3\pi/2, 2\pi]$

decreasing on $[0, \pi/2] \cup [\pi, 3\pi/2]$

13) $T(t) = 3t^3 - 18t$



$T'(t) = 9t^2 - 18$

$T''(t) = 18t = 0$

$\Rightarrow t = 0$

$t=0$ is an inflection point, concave up for $[0, \infty)$

concave down for $(-\infty, 0]$

18) $G(x) = 24x^2 + 12\sin^2(x)$

$G'(x) = 48x + 12 \cdot (2)\sin(x)\cos(x)$

$= 48x + 24\sin(x)\cos(x)$

$G''(x) = 48 + 24\left[\frac{d}{dx}[\sin(x)]\cos(x) + \sin(x)\frac{d}{dx}[\cos(x)]\right]$

$= 48 + 24(\cos^2(x) - \sin^2(x))$

$\cos^2(x) + \sin^2(x) = 1$

$\cos^2(x) = 1 - \sin^2(x)$

$= 48 + 24((1 - \sin^2(x)) - \sin^2(x))$

$= 48 + 24(1 - 2\sin(x))$

$= 48 + 24 - 48\sin^2(x)$

$= 72 - 48\sin^2(x) > 0$

for all x

$0 \le \sin^2(x) \le 1$

$\Rightarrow G(x)$ is always concave up

no inflection points

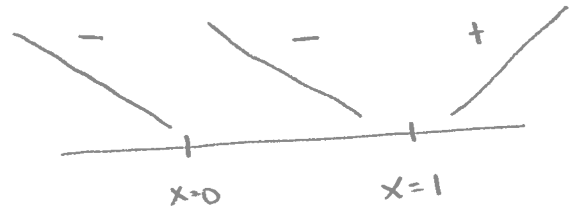
21) $g(x) = 3x^4 - 4x^3 + 2$

$g'(x) = 12x^3 - 12x^2$

$= 12x^2(x-1) = 0$

roots $x = 0, \quad x = 1$

double   single



decreasing for $x \le 1$

increasing for $x \ge 1$

$g'(2) = 12(2)^2(2-1) = 48 > 0$

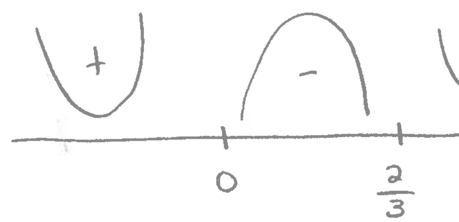
$g''(x) = 36x^2 - 24x$

$= 12x(3x-2) = 0$

roots $x = 0, \quad x = \frac{2}{3}$

both single

$g''(1) = 12(1)(3(1)-2) = 12 > 0$



concave up on $(-\infty, 0] \cup [\frac{2}{3}, \infty)$

concave down on $[0, \frac{2}{3}]$