Assignment 4

Section 2.3 : 5, 6, 32, 43

5) $y = 2x^{-2}$

$D_x[y] = D_x[2x^{-2}] = 2D_x[x^{-2}]$ Power rule

$= 2(-2x^{-3})$

$= \boxed{-4x^{-3}}$

6) $y = -3x^{-4}$

$D_x[y] = D_x[3x^{-4}]$ This time I will use quotient rule

$= 3D_x[x^{-4}] = 3D_x\left[\frac{1}{x^4}\right]$

$= 3\left[\frac{D_x[1]\cdot x^4 - D_x[x^4]\cdot 1}{x^8}\right]$ Power rule + Constant Rule

$= 3\left[\frac{0\cdot x^4 - 4x^3\cdot 1}{x^8}\right] = \frac{-12x^3}{x^8} = \boxed{-12x^{-5}}$

32) $y = (3x^2+2x)(x^4-3x+1)$

$D_x[y] = D_x\left[(3x^2+2x)(x^4-3x+1)\right]$ Product Rule

$= D_x[3x^2+2x]\cdot(x^4-3x+1) + (3x^2+2x)D_x[x^4-3x+1]$

$= (6x+2)(x^4-3x+1) + (3x^2+2x)(4x^3-3)$

$$= 6x(x^4 - 3x + 1) + 2(x^4 - 3x + 1) + 3x^2(4x^3 - 3) + 2x(4x^3 - 3)$$

$$= 6x^5 - 18x^2 + 6x + 2x^4 - 6x + 2 + 12x^5 - 9x^2 + 8x^3 - 6x$$

$$= \boxed{18x^5 + 2x^4 + 8x^3 - 27x^2 - 6x + 2}$$

43) $y = \dfrac{x^2 - x + 1}{x^2 + 1}$

$$D_x[y] = D_x\left[\frac{x^2 - x + 1}{x^2 + 1}\right]$$ Quotient Rule

$$= \frac{D_x[x^2 - x + 1] \cdot (x^2+1) - (x^2 - x + 1) D_x[x^2+1]}{(x^2+1)^2}$$

$$= \frac{(2x-1)(x^2+1) - (x^2 - x + 1)(2x)}{(x^2+1)^2}$$

$$= \frac{\cancel{2x^3} - x^2 + \cancel{2x} - 1 - (\cancel{2x^3} - 2x^2 + \cancel{2x})}{(x^2+1)^2}$$

$$= \boxed{\frac{x^2 - 1}{(x^2+1)^2}}$$

Section 2.4: 1, 14, 16

1) $y = 2\sin(x) + 3\cos(x)$

$$D_x(y) = D_x\left[2\sin(x) + 3\cos(x)\right]$$

$$= 2D_x\left[\sin(x)\right] + 3D_x\left[\cos(x)\right]$$

$$= \boxed{2\cos(x) - 3\sin(x)}$$

14) $y = \dfrac{1-\cos(x)}{x}$

$$D_x[y] = D_x\left[\frac{1-\cos(x)}{x}\right]$$

$$= \frac{D_x[1-\cos(x)]\cdot x - (1-\cos(x))D_x[x]}{x^2}$$

$$= \frac{\sin(x)\cdot x - (1-\cos(x))\cdot 1}{x^2} = \boxed{\frac{x\sin(x) + \cos(x) - 1}{x^2}}$$

16) $y = \dfrac{x\cos(x) + \sin(x)}{x^2+1}$

$$D_x[y] = D_x\left[\frac{x\cos(x) + \sin(x)}{x^2+1}\right]$$

$$= \frac{D_x[x\cos(x) + \sin(x)](x^2+1) - (x\cos(x) + \sin(x))D_x[x^2+1]}{(x^2+1)^2}$$

Note: $D_x[x\cos(x)+\sin(x)] = D_x[x\cos(x)] + D_x[\sin(x)]$

$= D_x[x\cos(x)] + \cos(x)$

$D_x[x\cos(x)] = D_x[x]\cos(x) + D_x[\cos(x)]\,x$

$= \cos(x) - x\sin(x)$

$$\Rightarrow D_x[y] = \frac{(2\cos(x) - x\sin(x))(x^2+1) - (x\cos(x)+\sin(x))(2x)}{(x^2+1)^2}$$

$$= \frac{\cancel{2x^2\cos(x)} - x^3\sin(x) + 2\cos(x) - x\sin(x) - \cancel{2x^2\cos(x)} + 2x\sin(x)}{(x^2+1)^2}$$

$$= \boxed{\frac{-x^3\sin(x) + x\sin(x) + 2\cos(x)}{(x^2+1)^2}}$$

Section 2.5: 3, 17, 20, 38

3) $y = (3-2x)^5$

$D_x[y] = D_x\left[(3-2x)^5\right] = f'(g(x))\cdot g'(x)$

inside $g(x) = 3-2x$ $\qquad g'(x) = -2$

outside $f(x) = x^5$ $\qquad f'(x) = 5x^4$

then $f(g(x)) = (3-2x)^5$ ✓ $\qquad f'(g(x)) = 5(3-2x)^4$

$\Rightarrow D_x[y] = 5(3-2x)^4(-2) = \boxed{-10(3-2x)^4}$

17) $y = (3x-2)^2 (3-x^2)^2$

$$D_x[y] = D_x\left[(3x-2)^2(3-x^2)^2\right]$$

$$= D_x\left[(3x-2)^2\right]\cdot(3-x^2)^2 + (3x-2)^2 D_x\left[(3-x^2)^2\right] \quad \text{(product rule)}$$

Note: $D_x\left[(3x-2)^2\right] = f'(g(x))\cdot g'(x) = 3\cdot 2\cdot(3x-2) = 6(3x-2)$

inside $g(x) = 3x-2$ $\qquad g'(x) = 3$

outside $f(x) = x^2$ $\qquad f'(x) = 2x$

$f(g(x)) = (3x-2)^2$ ✓ $\qquad f'(g(x)) = 2(3x-2)$

$$D_x\left[(3-x^2)^2\right] = f'(g(x))\,g'(x) = 2(3-x^2)(-2x) = -4x(3-x^2)$$

inside $g(x) = 3-x^2$ $\qquad g'(x) = -2x$

outside $f(x) = x^2$ $\qquad f'(x) = 2x$

$f(g(x)) = (3-x^2)^2$ ✓ $\qquad f'(g(x)) = 2(3-x^2)$

$$\Rightarrow D_x[y] = \big(6(3x-2)\big)(3-x^2)^2 + (3x-2)^2\big(-4x(3-x^2)\big)$$

$$= 6(3x-2)(3-x^2)^2 - 4x(3x-2)^2(3-x^2)$$

$$= 2(3x-2)(3-x^2)\left[3(3-x^2) - 2x(3x-2)\right]$$

$$= \boxed{2(3x-2)(3-x^2)(-9x^2+4x+9)}$$

20) $y = \left[\dfrac{2x-3}{(x^2+4)^2} \right]$

$$D_x[y] = \frac{D_x[2x-3](x^2+4)^2 - D_x[(x^2+4)^2](2x-3)}{(x^2+4)^4}$$ (Quotient Rule)

Note: $D_x[(x^2+4)^2] = f'(g(x))g'(x) = 2(x^2+4)(2x) = 4x(x^2+4)$

inside: $g(x) = x^2+4$ $\quad g'(x) = 2x$

outside $f(x) = x^2$ $\quad f'(x) = 2x$

$f(g(x)) = (x^2+4)^2$ ✓ $\quad f'(g(x)) = 2(x^2+4)$

$$\Rightarrow D_x[y] = \frac{2(x^2+4)^2 - 4x(x^2+4)(2x-3)}{(x^2+4)^4}$$

$$= \frac{2(x^2+4)\left((x^2+4) - 2x(2x-3)\right)}{(x^2+4)^4}$$

$$= \frac{2(x^2+4-4x^2+6x)}{(x^2+4)^3} = \frac{2(-3x^2+6x+4)}{x^2+4}$$

$$= \boxed{\frac{-6x^2+12x+8}{(x^2+4)^3}}$$

38) $D_x\left[x\sin^2(2x)\right]$

$= D_x[x]\sin^2(2x) + x\cdot D_x\left[\sin^2(2x)\right]$

Note: $D_x\left[\sin^2(2x)\right]$

inside $g(x) = 2x$ $\qquad g'(x) = 2$

outside $f(x) = \sin^2(x)$ $\qquad f'(x) = D_x\left[\sin^2(x)\right]$ *

$f(g(x)) = \sin^2(2x)$ ✓ $\qquad f'(g(x)) = 2\sin(2x)\cos(2x)$

* $D_x\left[\sin^2(x)\right] = D_x\left[(\sin(x))^2\right] = f'(g(x))g'(x) = 2\sin(x)\cos(x)$

inside $g(x) = \sin(x)$ $\qquad g'(x) = \cos(x)$

outside $f(x) = x^2$ $\qquad f'(x) = 2x$

$f(g(x)) = \sin^2(x)$ ✓ $\qquad f'(g(x)) = 2\sin(x)$

$\Rightarrow D_x\left[\sin^2(2x)\right] = 2\sin(2x)\cos(2x)\,(2) = 4\sin(2x)\cos(2x)$

$\Rightarrow D_x\left[x\sin^2(2x)\right] = \sin^2(2x) + x\left(4\sin(2x)\cos(2x)\right)$

$= \sin^2(2x) + 4x\sin(2x)\cos(2x)$

$= \boxed{\sin(2x)\left(\sin(2x) + 4x\cos(2x)\right)}$

Section 2.6: 1, 13, 14

1) $y = x^3 + 3x^2 + 6x$

$$\frac{dy}{dx} = 3x^2 + 6x + 6$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[3x^2 + 6x + 6\right]$$
$$= 6x + 6$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right] = \frac{d}{dx}\left[6x + 6\right]$$
$$= \boxed{6}$$

13) $f(\theta) = \cos(\theta\pi)^{-2}$

$f'(\theta) = \frac{d}{d\theta}\left[\cos(\theta\pi)^{-2}\right]$ (chain rule)

inside $g(\theta) = \theta\pi$ $g'(\theta) = \pi$

outside $f(x) = (\cos(x))^{-2}$ $f'(x) = \frac{d}{dx}\left[\cos(x)^{-2}\right]$ (chain rule)

$f(g(\theta)) = \cos(\theta\pi)^{-2}$ ✓

$$D_x\left[\cos(x)^{-2}\right] = f'(g(x))g'(x) = -2\cos(x)^{-3}(-\sin(x)) = \frac{2\sin x}{\cos^3(x)}$$

inside $g(x) = \cos(x)$ $g'(x) = -\sin(x)$

outside $f(x) = x^{-2}$ $f'(x) = -2x^{-3}$

$f(g(x)) = \cos(x)^{-2}$ ✓ $f'(g(x)) = -2\cos(x)^{-3}$

$$\Rightarrow \frac{d}{d\theta}\left[\cos(\theta\pi)^{-2}\right] = \frac{2\sin(\theta\pi)}{\cos^3(\theta\pi)} \cdot \pi$$

$$= 2\pi \tan(\theta\pi)\cos(\theta\pi)^{-2}$$

$$f''(\theta) = \frac{d}{d\theta}\left[f'(\theta)\right] = \frac{d}{d\theta}\left[2\pi \tan(\theta\pi)\cos(\theta\pi)^{-2}\right] \quad \text{(product rule)}$$

$$= 2\pi\left[D_\theta\left[\tan(\theta\pi)\right]\cos(\theta\pi)^{-2} + \tan(\theta\pi)\, D_\theta\left[\cos(\theta\pi)^{-2}\right]\right]$$

$$D_\theta\left[\tan(\theta\pi)\right] = f'(g(\theta))\,g'(\theta) = \pi\sec^2(\theta\pi)$$

inside $g(\theta) = \theta\pi$ $\qquad g'(\theta) = \pi$

outside $f(x) = \tan(x)$ $\qquad f'(x) = \sec^2(x)$

$f(g(\theta)) = \tan(\theta\pi)$ ✓ $\qquad f'(g(\theta)) = \sec^2(\theta\pi)$

$$\Rightarrow f''(\theta) = 2\pi\left[\left(\pi\sec^2(\theta\pi)\cos(\theta\pi)^{-2}\right) + \tan(\theta\pi)\left(2\pi\tan(\theta\pi)\cos(\theta\pi)^{-2}\right)\right]$$

$$= 2\pi^2\left(\frac{1}{\cos^2(\theta\pi)}\right)\left(\frac{1}{\cos^2(\theta\pi)}\right) + 2\pi\left(\frac{\sin^2(\theta\pi)}{\cos^2(\theta\pi)}\right)\left(\frac{1}{\cos^2(\theta\pi)}\right)$$

$$= \frac{2\pi^2}{\cos^4(\theta\pi)} + \frac{2\pi\sin^2(\theta\pi)}{\cos^4(\theta\pi)}$$

$$= \frac{2\pi\left(\pi + \sin^2(\theta\pi)\right)}{\cos^4(\theta\pi)}$$

$$f''(2) = \frac{2\pi\left(\pi + \sin^2(2\pi)\right)}{\cos^4(2\pi)} = \frac{2\pi(\pi + 0)}{1^4} = \boxed{2\pi^2}$$

14) $f(t) = t \sin\left(\frac{\pi}{t}\right)$

$$f'(t) = D_t[t] \sin\left(\frac{\pi}{t}\right) + D_t\left[\sin(\pi/t)\right] \cdot t \qquad \text{(product rule)}$$

$$D_t\left[\sin(\pi/t)\right] = f'(g(t)) \cdot g'(t) = \cos(\pi/t)\left(-\pi t^{-2}\right)$$

inside $\quad g(t) = \pi t^{-1} \qquad g'(t) = -\pi t^{-2}$

outside $\quad f(x) = \sin(x) \qquad f'(x) = \cos(x)$

$f(g(x)) = \sin\left(\frac{\pi}{t}\right)$ ✓ $\qquad f'(g(t)) = \cos\left(\frac{\pi}{t}\right)$

$$\Rightarrow f'(t) = 1 \cdot \sin(\pi/t) + \left(-\pi t^{-2}\right)\left(\cos(\pi/t)\right) t$$

$$= \sin(\pi/t) - \pi t^{-1} \cos(\pi/t)$$

$$f''(t) = D_t\left[\sin(\pi/t)\right] - \pi D_t\left[\frac{\cos(\pi/t)}{t}\right] \qquad \text{(Quotient Rule)}$$

$$= -\pi t^{-2} \cos\left(\frac{\pi}{t}\right) - \pi\left[\frac{D_t[\cos(\pi/t)]\, t - D_t[t] \cos(\pi/t)}{t^2}\right]$$

$$D_t\left[\cos(\pi/t)\right] = f'(g(t))\, g'(t) = -\sin(\pi/t)\left(-\pi t^{-2}\right) = \frac{\pi}{t^2} \sin(\pi/t)$$

inside $= \quad g(t) = \pi t^{-1} \qquad g'(t) = -\pi t^{-2}$

outside $= \quad f(x) = \cos(x) \qquad f'(x) = -\sin(x)$

$f(g(t)) = \cos(\pi/t)$ ✓ $\qquad f'(g(t)) = -\sin(\pi/t)$

$$\Rightarrow f''(t) = \frac{-\pi}{t^2} \cos(\pi/t) - \pi\left[\frac{\left(\frac{\pi}{t^2} \sin(\pi/t) \cdot t - \cos(\pi/t)\right)}{t^2}\right]$$

$$= \frac{-\pi \cos(\pi/t)}{t^2} - \frac{\pi}{t^2}\left[\frac{\pi}{t} \sin(\pi/t) - \cos(\pi/t)\right]$$

$$f''(2) = \frac{-\pi}{4} \cos(\pi/2) - \frac{\pi}{4}\left[\frac{\pi}{2} \sin(\pi/2) - \cos(\pi/2)\right]$$

$$f''(2) = -\frac{\pi}{4}(0) - \frac{\pi}{4}\left[\frac{\pi}{2}(1) - 0\right]$$

$$= \boxed{-\frac{\pi^2}{8}}$$

Section 2.7 : 1, 4, 10, 15

1) $y^2 - x^2 = 1$

$$D_x\left[y^2 - x^2\right] = D_x[1]$$

$$\underbrace{D_x\left[y^2\right]} - D_x\left[x^2\right] = 0$$

inside $g(y) = y$ $\quad \frac{d}{dx} g = \frac{dy}{dx}$

outside $f(x) = x^2$ $\quad f'(x) = 2x$ $\quad \Rightarrow D_x[y] = 2y \frac{dy}{dx}$

$f(g(y)) = y^2$ ✓ $\quad f'(g) = 2y$

$$\Rightarrow \quad 2y\frac{dy}{dx} - 2x = 0$$

$$\frac{dy}{dx} = \frac{2x}{2y} \quad \Rightarrow \quad \boxed{\frac{dy}{dx} = \frac{x}{y}}$$

4) $x^2 + \alpha^2 y^2 = 4\alpha^2$ where α is a constant

$$D_x\left[x^2\right] + D_x\left[\alpha^2 y^2\right] = D_x\left[4\alpha^2\right]$$

$$D_x\left[x^2\right] + \alpha^2 D_x\left[y^2\right] = 0$$

$$2x + \alpha^2\left(2y\frac{dy}{dx}\right) = 0$$

$$2\alpha^2 y\frac{dy}{dx} = -2x$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{2\alpha^2 y}}$$

10) $x\sqrt{y+1} = xy + 1$

$$D_x\left[x(y+1)^{\frac{1}{2}}\right] = D_x[xy] + D_x[1]$$

$$D_x[x](y+1)^{\frac{1}{2}} + x\,D_x\left[(y+1)^{\frac{1}{2}}\right] = D_x[x]\,y + x\,D_x[y] + 0$$

$$D_x\left[(y+1)^{\frac{1}{2}}\right] = D_x\left[f(g(y))\right] = \frac{df}{dg}\frac{dg}{dx} = f'(g)\frac{dg}{dx} = \frac{1}{2}(y+1)^{-\frac{1}{2}}\frac{dy}{dx}$$

inside $g(y) = y+1$ $\qquad \frac{d}{dx}g(y) = \frac{d}{dx}[y+1] = \frac{dy}{dx}$

outside $f(x) = x^{\frac{1}{2}}$ $\qquad f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f(g(y)) = (y+1)^{\frac{1}{2}}$ ✓ $\qquad f'(g) = \frac{1}{2}(y+1)^{-\frac{1}{2}}$

$$\Rightarrow \quad 1\cdot(y+1)^{\frac{1}{2}} + x\left(\frac{1}{2}(y+1)^{-\frac{1}{2}}\frac{dy}{dx}\right) = 1\cdot y + x\cdot\frac{dy}{dx}$$

$$\frac{x}{2\sqrt{y+1}}\frac{dy}{dx} - x\frac{dy}{dx} = y - \sqrt{y+1}$$

$$x\left(\frac{1}{2\sqrt{y+1}} - 1\right)\frac{dy}{dx} = y - \sqrt{y+1}$$

$$\frac{dy}{dx} = \frac{y-\sqrt{y+1}}{x\left(\frac{1}{2\sqrt{y+1}} - 1\right)}$$

$$\frac{dy}{dx} = \frac{y-\sqrt{y+1}}{x\left(\frac{1}{2\sqrt{y+1}} - \frac{2\sqrt{y+1}}{2\sqrt{y+1}}\right)} = \frac{y-\sqrt{y+1}}{x\left(\frac{1-2\sqrt{y+1}}{2\sqrt{y+1}}\right)}$$

$$\boxed{\frac{dy}{dx} = \frac{2\sqrt{y+1}\left(y-\sqrt{y+1}\right)}{x\left(1-2\sqrt{y+1}\right)}}$$

$$15)\quad \sin(xy) = y \quad ; \quad \begin{matrix}(x,y)\\ \left(\frac{\pi}{2}, 1\right)\end{matrix}$$

(13)

$$\underbrace{D_x\left[\sin(xy)\right]}= D_x[y]$$

inside $g(x,y) = xy$ $\qquad \frac{d}{dx}[g] = D_x[x]\,y + D_x[y]\,x = y + x\frac{dy}{dx}$

outside $f(x) = \sin(x)$ $\qquad f'(x) = \cos(x)$

$f(g(x,y)) = \sin(xy)$ ✓ $\qquad f'(g) = \cos(xy)$

$$\Rightarrow D_x[\sin(xy)] = \left(y + x\frac{dy}{dx}\right)\cos(xy)$$

$$\Rightarrow \left(y + x\frac{dy}{dx}\right)\cos(xy) = \frac{dy}{dx}$$

$$y\cos(xy) + x\cos(xy)\frac{dy}{dx} = \frac{dy}{dx}$$

$$y\cos(xy) = \frac{dy}{dx} - x\cos(xy)\frac{dy}{dx}$$

$$y\cos(xy) = \left(1 - x\cos(xy)\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$$

$$\Rightarrow \frac{dy}{dx} \text{ at } \left(\frac{\pi}{2}, 1\right) = \frac{1\cdot\cos\left(\frac{\pi}{2}\cdot 1\right)}{1 - \frac{\pi}{2}\cos\left(\frac{\pi}{2}\cdot 1\right)}$$

$$= \frac{\cos\left(\frac{\pi}{2}\right)}{1 - \frac{\pi}{2}\cos\left(\frac{\pi}{2}\right)} = \frac{0}{1 - 0} = \boxed{0}$$

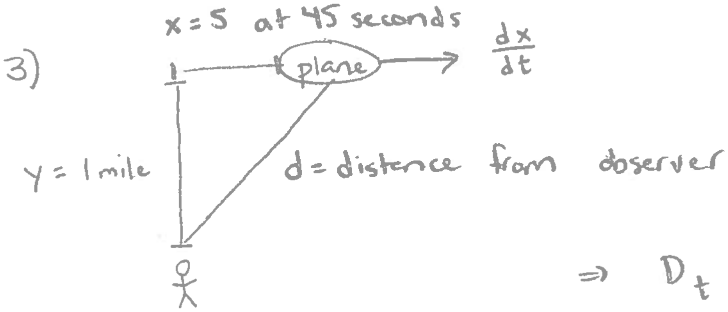
$$\Rightarrow y - y_1 = \frac{dy}{dx}(x - x_1)$$

$$y - 1 = 0\left(x - \frac{\pi}{2}\right) \quad \Rightarrow \quad \boxed{y = 1} \quad \text{equation of tangent line}$$

$$y - 1 = 0$$

Section 2.8: 3, 5, 13, 16

3)



$d = \sqrt{x^2 + y^2}$

$\Rightarrow D_t[d] = D_t\left[(x^2+y^2)^{\frac{1}{2}}\right]$

$$D_t\left[(x^2+y^2)^{\frac{1}{2}}\right] = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2\sqrt{x^2+y^2}} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$$

inside $g = (x^2+y^2)$ $\qquad \frac{dg}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

outside $f = x^{\frac{1}{2}}$ $\qquad f' = \frac{1}{2}x^{-\frac{1}{2}} \Rightarrow f'(g) = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}$

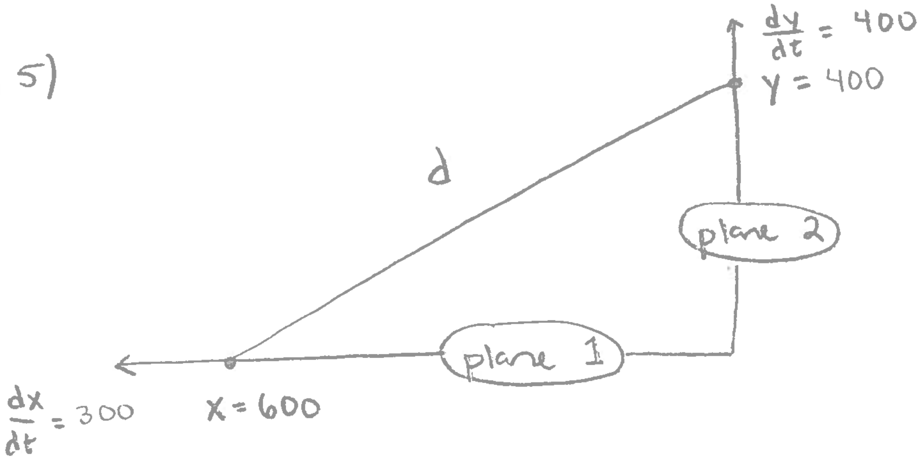
$$\Rightarrow D_t[d] = \frac{x\left(\frac{dx}{dt}\right) + y\left(\frac{dy}{dt}\right)}{\sqrt{x^2+y^2}}$$

$x = 5$ $\qquad \frac{dx}{dt} = 400$

$y = 1$ $\qquad \frac{dy}{dt} = 0$

$$= \frac{5(400) + (1)(0)}{\sqrt{5^2 + 1^2}} = \frac{2000}{\sqrt{26}} \approx \boxed{392 \text{ mph}}$$

5)



from problem 3 we have $\qquad D_t[d] = \dfrac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$

$x = 600$

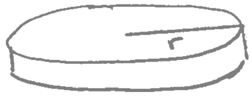
$y = 400$

$\frac{dx}{dt} = 300$

$\frac{dy}{dt} = 400$

$$= \frac{(600)(300) + (400)(400)}{\sqrt{600^2 + 400^2}} = \frac{340{,}000}{\sqrt{520{,}000}} \approx \boxed{471.49 \text{ mph}}$$

13)



$r = 8.1$ in

$\frac{dr}{dt} = 0.02 \ \frac{in}{sec}$

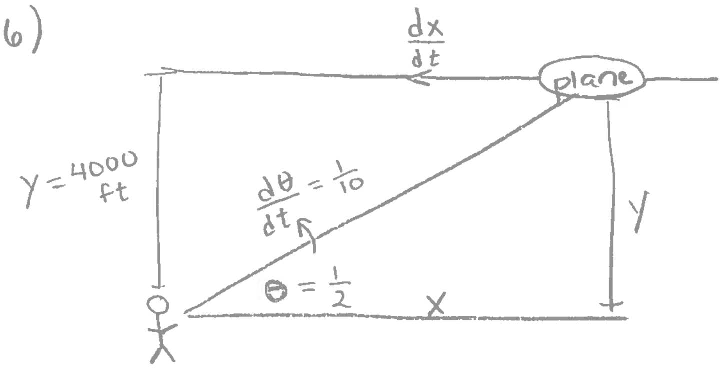
$$A = \pi r^2$$

$$D_t[A] = \pi D_t[r^2] = \pi\left(2r\frac{dr}{dt}\right) = 2\pi r\frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi(8.1)(0.02)$$

$$\boxed{\frac{dA}{dt} \approx 1.02 \ \frac{in^2}{sec}}$$

16)



Need to relate θ, x, y

$$\tan\theta = \frac{y}{x} \Rightarrow x = \frac{y}{\tan\theta}$$

$$\Rightarrow D_t[\tan\theta] = D_t\left[\frac{y}{x}\right]$$

$$\sec^2\theta \ \frac{d\theta}{dt} = \frac{D_t[y]x - D_t[x]y}{x^2} = \frac{\frac{dy}{dt}x - \frac{dx}{dt}y}{x^2}$$

$x = \frac{4000}{\tan(1/2)}$

$y = 4000$

$\theta = \frac{1}{2}$

$\frac{dy}{dt} = 0$

$\frac{d\theta}{dt} = \frac{1}{10}$

$$\Rightarrow \frac{dy}{dt}x - \frac{dx}{dt}y = x^2\sec^2(\theta)\frac{d\theta}{dt}$$

$$-\frac{dx}{dt}y = x^2\sec^2(\theta)\frac{d\theta}{dt} - \frac{dy}{dt}x$$

$$\frac{dx}{dt} = \frac{x^2\sec^2(\theta)\frac{d\theta}{dt} - x\frac{dy}{dt}}{-y}$$

$$\frac{dx}{dt} = \frac{\left(\frac{4000}{\tan(1/2)}\right)^2\sec^2(1/2)\left(\frac{1}{10}\right) - \left(\frac{4000}{\tan(1/2)}\right)(0)}{-4000} = -1740 \Rightarrow \boxed{\text{speed} = 1740 \ \frac{ft}{sec}}$$